1972

Evaluation of minimum aircraft flying speed by digital simulation.

Saul, Carlton Wayne.

Monterey, California. Naval Postgraduate School

EVALUATION OF MINIMUM AIRCRAFT FLYING SPEED BY DIGITAL SIMULATION

Carlton Wayne Saul

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

Evaluation of Minimum Aircraft Flying
Speed by Digital Simulation

by

Carlton Wayne Saul

Thesis Advisor: L.V. Schmidt

June 1972

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Speed by Digital Simulation

by

Carlton Wayne Saul
Lieutenant, United States Navy
B. S., Lynchburg College, 1964

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1972

ABSTRACT

Aircraft minimum flying speed, as determined by actual flight test, is published in aircraft handbooks for pilot guidance. The test flight results are used to determine and confirm take-off and landing speeds, field lengths, left-hand portion of the maneuvering envelopes (V-n diagram), etc. Determination of the absolute minimum flying speed of an aircraft on the other hand, has not been of prime importance in flight test.

In the present analysis digital simulation allowed the systematic study of not only the minimum flying speed as defined by Federal Aviation Regulations but also the absolute minimum flying speed attainable in steady, unaccelerated flight. The study included such effects as deceleration rate, rate of change of elevator angle, aircraft weight and pitch moment of inertia.

It was found for an assumed light-weight fighter aircraft that the absolute minimum flying speed was approximately 20 knots less than the FAR minimum flying speed. Moreover the FAR minimum flying speeds tended to be quite sensitive to rate of change of elevator angle.

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SYMBOLS

$a_{n_{fp}}$	normal flight path acceleration, ft/sec^2
B	pitch moment of inertia, slug-ft^2
C	location of the aircraft center of mass
$\bar{c}$	mean aerodynamic chord
C_D	drag coefficient, drag/$\bar{q}$S
C_L	lift coefficient, lift/$\bar{q}$S
$C_{L_{max}}$	maximum lift coefficient
C_L'	apparent lift coefficient, weight/$\bar{q}$S
$C'_{L_{max}}$	maximum apparent lift coefficient
$C_L(\alpha)$	static lift coefficient as a function of angle of attack
$C_{L_{\delta_e}}$	rate of change of lift coefficient for a change in elevator angle
$C_{L_{\dot{\theta}}}$	rate of change of lift coefficient for a change in pitch velocity.
C_M	pitching moment coefficient, moment/$\bar{q}S\bar{c}$
$C_M(\alpha)$	static pitching moment coefficient as a function of angle of attack
$C_{M_{\dot{\alpha}}}$	angle of attack damping derivative, $\dfrac{\partial C_m}{\partial\left(\frac{\dot{\alpha}c}{2V}\right)}$ (rad^{-1})
$C_{M_{\delta_e}}$	pitching moment coefficient derivative for elevator deflection
C_{M_q}	pitch damping derivative, $\dfrac{\partial C_m}{\partial\left(\frac{\dot{\theta}c}{2V}\right)}$ (rad^{-1})
C_T	thrust coefficient, thrust/$\bar{q}$S
C_X	longitudinal aerodynamic force coefficient, body axis
C_Z	vertical aerodynamic force coefficient, body axis

D	product of inertia $\iint yz\,dm$, slug-ft^2
δ_e	elevator angle, deg.
$\dot{\delta}_e$	rate of change of elevator angle, deg/sec.
e_t	local truncation error for Runge-Kutta algorithm
F	product of inertia $\iint xy\,dm$, slug-ft^2
$\overline{F}$	resultant external force vector relative to aircraft center of mass
$\overline{G}$	resultant external moment vector about the aircraft center of mass
g	scalar acceleration due to gravity, ft/sec^2
$\overline{h}$	angular momentum vector relative to aircraft center of mass
[L,M,N]	scalar components of $\overline{G}$; rolling, pitching and yawing moments, ft-lb.
[P,Q,R]	scalar components of $\overline{W}$; rolling, pitching and yawing velocities, rad/sec.
$\dot{Q}$	rate of change of pitching velocity
$\overline{q}$	dynamic pressure, lb/ft^2
$\overline{q}_s$	dynamic pressure at minimum flying speed/stall speed, lb/ft^2
S	wing area, ft^2
T	thrust, lb.
[u,v,w]	scalar components of $\overline{V}_c$, relative to body axis system, ft/sec.
$\dot{U}$	rate of change of V, ft/sec^2
$\dot{V}$	rate of change of V_c, as determined by FAR method
$\dot{V}_n$	rate of change of V_c, as determined by altitude break method
V_c	magnitude of resultant linear velocity of aircraft center mass, ft/sec.
$\overline{V}_c$	resultant velocity vector of aircraft center of mass, ft/sec

$V_{c_{min}}$ minimum flying speed as determined by C_L method, ft/sec.

$V_{c'_{min}}$ minimum flying speed as determined by C_L' method, ft/sec.

$V_{h_{min}}$ minimum flying speed as determined by altitude break method, ft/sec.

V_{min} minimum flying speed as determined by constant deceleration rate, ft/sec.

W aircraft weight, lb.

$\dot{W}$ rate of change of W, scalar component of $\overline{V_c}$, ft/sec^2.

[x,y,z] cartesian coordinate axes notation for body axis system, orgin located at aircraft center of mass.

[x',y',z'] cartesian coordinate axes notation for stability axes system, orgin located at aircraft center of mass.

[X,Y,Z] components of resultant aerodynamic force acting on aircraft, lb.

$Y_{n+1,1}$ variable in Runge-Kutta algorithm obtained by intergrating between two points, X_n and X_{n+1}, with step size h_1.

$Y_{n+1,2}$ variable in Runge-Kutta algorithm obtained by intergrating between two points, X_n and X_{n+1}, with step size h_2 where $h_2 = h_1/2$

α angle of attack, deg.

$\dot{\alpha}$ rate of change of angle of attack, deg/sec.

θ aircraft pitch angle, angle between horizontal reference and the X axis of the body axis system, deg.

$\dot{\theta}$ rate of change of pitch angle, deg/sec.

ρ air density, slugs/ft^3

$[\Psi,\Theta,\Phi]$ Euler angles, rad.

$\overline{\omega}$ angular velocity vector of the aircraft, rad/sec.

I. INTRODUCTION

Minimum flying speed and or stalling speed is defined by Federal Aviation Regulations (FAR) [Ref. 1] as the minimum steady speed at which the aircraft remains controllable. The Naval Test Pilot School Flight Test Manual [Ref. 2] defines minimum flying speed as the minimum steady airspeed attainable in unaccelerated flight or the minimum usable airspeed. The actual determination of minimum flying speed is always accomplished through flight tests. Methods for theoretical predictions are empirical, based upon wind-tunnel results, experience, etc.

FAR specified that stalls would be demonstrated by trimming the aircraft at 130 percent of the estimated minimum flying speed and decelerating at a constant rate until a minimum speed was obtained. The certified stall speed would correspond to a deceleration rate of 1 knot/sec. To facilitate the test pilot's task, Ref. 1 and 2 defined aircraft characteristics and parameters indicative of minimum flying speed. Typical indications were: longitudinal, directional or lateral divergence, excessive altitude loss, loss of control/effectiveness, etc. Data analysis methods and restrictions on demonstration technique, other than rate of change of airspeed, were not specified by Ref. 1.

Additional flight test techniques have been proposed and evaluated by the National Aeronautics and Space Administration [Refs. 3 and 4]. These studies, through flight tests, evaluated three demonstration techniques: FAR, 1-g break and constant-rate-of-climb. Three methods of data-analysis were used with the FAR demonstration technique to

determine minimum flying speed: constant deceleration, C_L^1 and C_L^2 methods. The minimum flying speeds obtained by the three techniques were corrected to a deceleration rate of 1 knot/sec in accordance with Ref. 1.

The study described herein applied numerical methods to simulate flight test data in an attempt to define an absolute minimum flying speed. Five data-analysis methods were used. The speed was an absolute minimum obtainable for steady, unaccelerated flight when the FAR correction to a 1 knot/sec. deceleration rate was disallowed. The absolute minimum flying speeds were compared with FAR certified minimum flying speeds, i.e., corrected to 1 knot/sec. deceleration rate.

"Flight test data" for the F-94A, a small single-engine jet fighter, was obtained from a computer program of the non-linear equations of aircraft motion. FORTRAN IV language and the Naval Postgraduate School IBM 360 digital computer were utilized. The computer program provided numerous options for trimming and flying the aircraft, including variations of thrust, weight, rate of change of elevator angle and flight orientation, i.e., climbing, level or descending flight.

For this study, the aircraft was trimmed in steady, descending flight at 130 percent of the estimated minimum flying speed and decelerated by decreasing the elevator angle at a constant rate. Two values for thrust were used to trim and stall the aircraft: $T/W \leq 0.02$ and $T/W = 0.11$.

The effects on minimum flying speed of thrust, aircraft weight, rate of change of elevator angle and aircraft moment of inertia about the lateral axis were also studied.

The purpose of this study was to investigate the relationships of the aircraft minimum flying speeds as determined by the different data-analysis methods.

II. EXPERIMENTAL METHODS

A. BASIC APPROACH

The study of minimum flying speed was restricted to aircraft motion in the XZ plane: i.e., to aircraft motion along the X and Z axes and pitching moments about the Y axis. It was assumed that no adverse lateral or directional traits occurred while decelerating to the minimum flying speed.

Non-linear equations of aircraft motion with three degrees-of-freedom were derived from Euler's equations of motion, Appendix A. Extensive use was made of Etkin [Ref. 5]. The equations were set up for input of non-linear aerodynamic data, by means of a table look-up procedure.

A computer program was written in FORTRAN IV language for the Naval Postgraduate School IBM 360 digital computer, Appendix B. The program employed a fourth-order Runge-Kutta algorithm in solving the equations of motion. Real-time was used in evaluating the aircraft motion from approximately 1.3 V_{min} to V_{min}. Typical time histories of the aircraft parameters were plotted in Figure 1. Figure 2 contains summary plots of V_{min} as a function of deceleration rate for different rates of change of elevator angle. The thrust to weight ration (T/W) was 0.02.

B. TEST AIRCRAFT

The F-94A, a small single-engine jet fighter, was utilized as the test aircraft.

Reference aerodynamic data were obtained from Blakelock [Ref. 5] in the form of linear stability derivatives for all terms except those

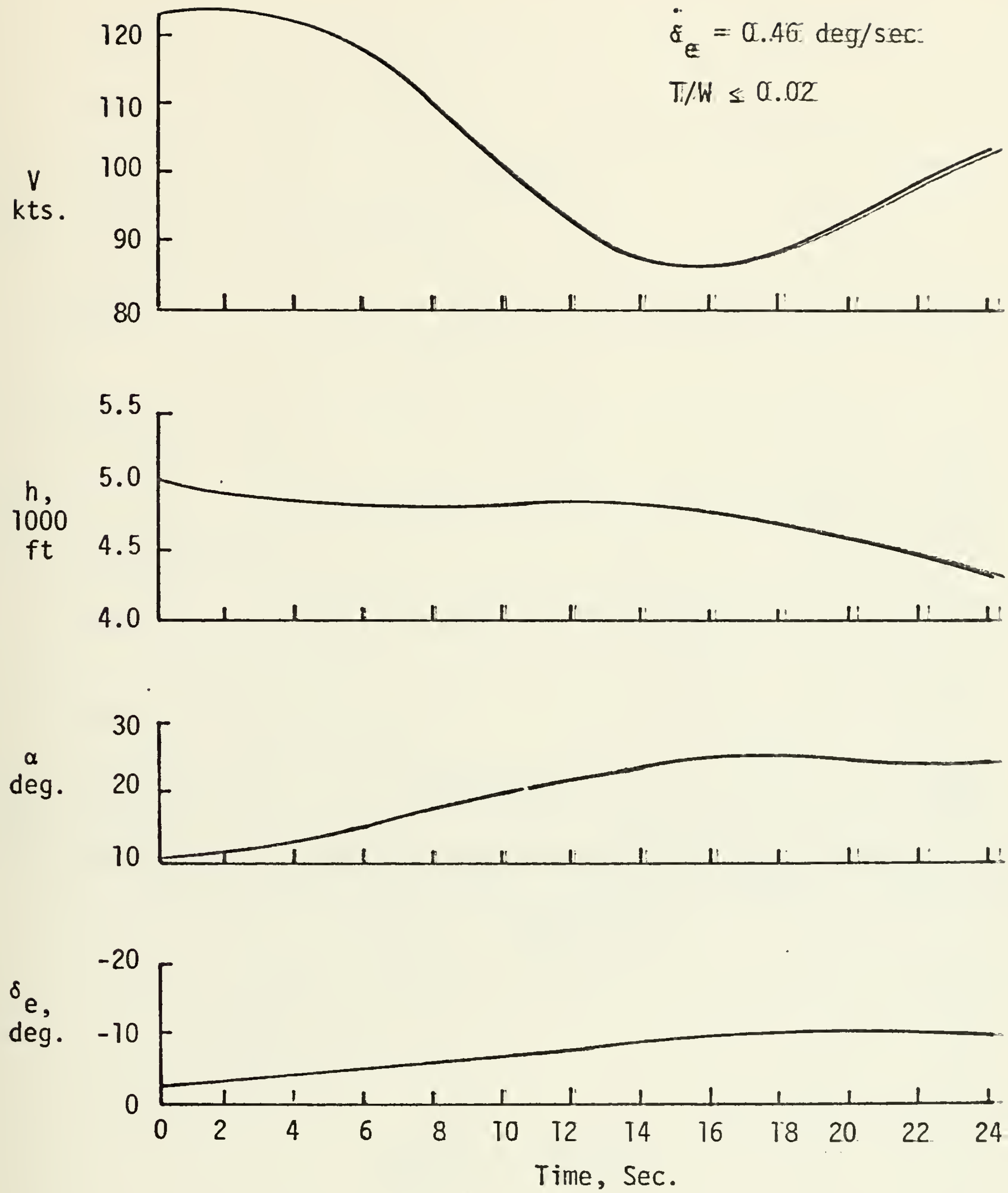

Figure 1. Typical Time Histories of Aircraft Parameters.

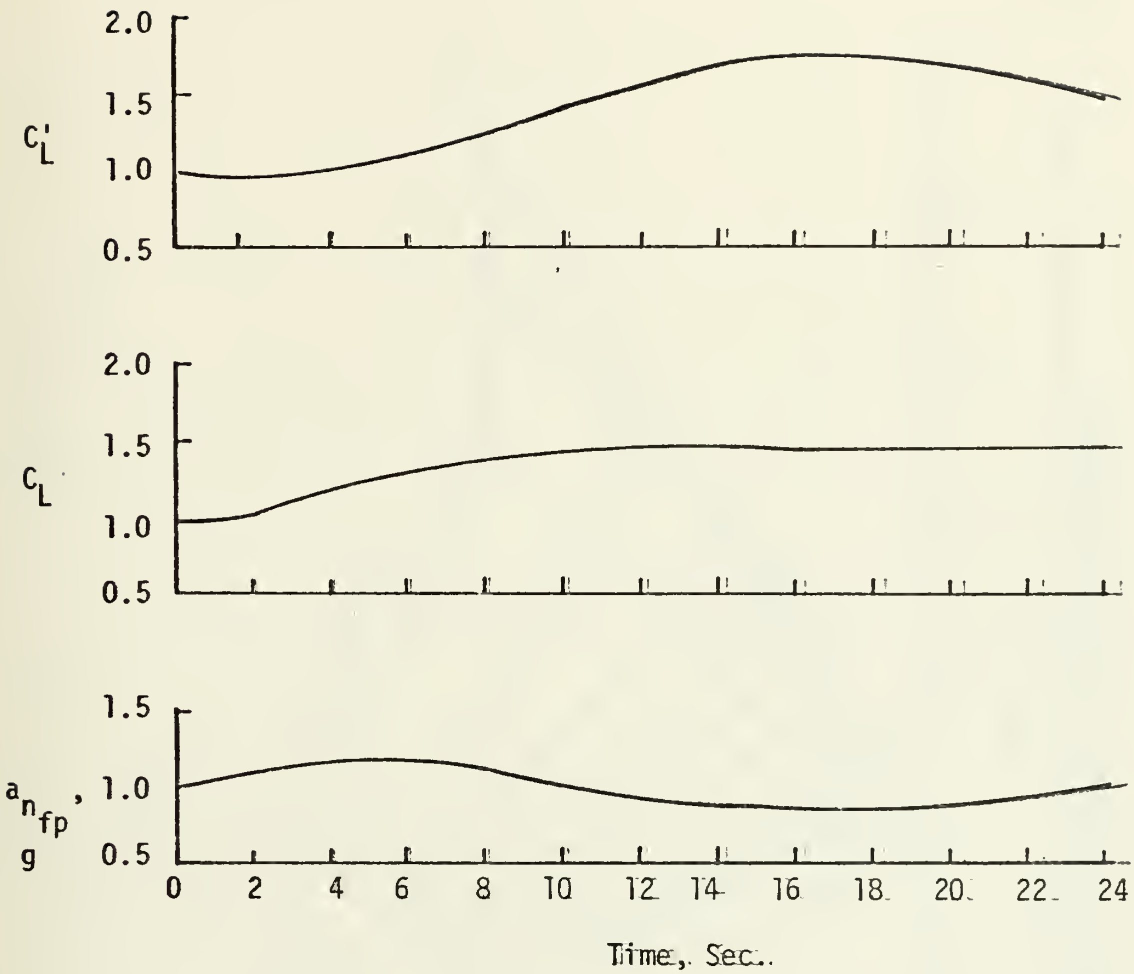

Figure 1. (Cont'd)

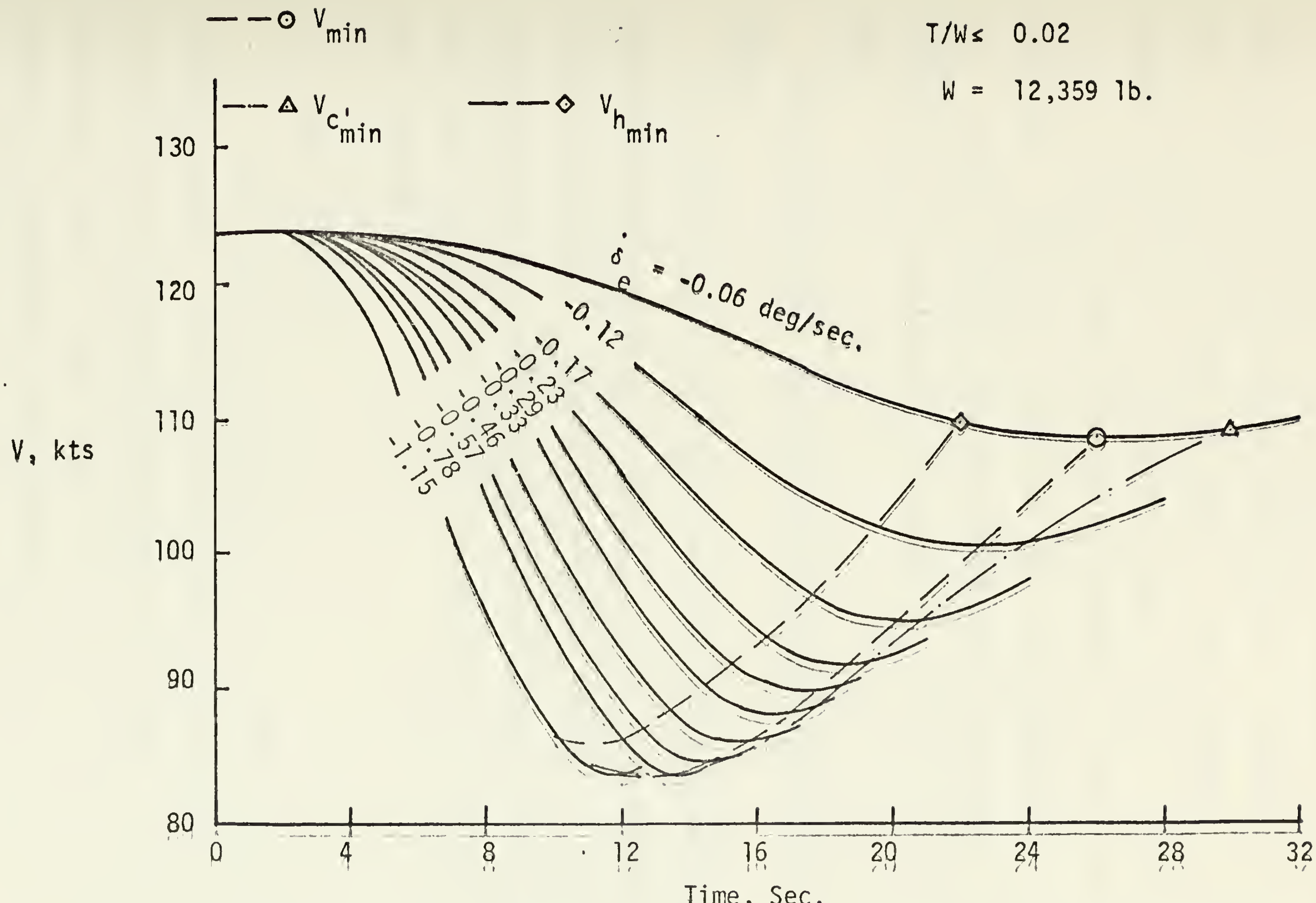

Figure 2. Time History Trends of Aircraft Deceleration, Elevator Varried From -0.06 to -1.15 deg/sec. To Obtain Plots, $\dot{\delta}_e$ Constant During Each Maneuver.

depending upon angle of attack. The variation of the complete aircraft C_L, C_D and C_M was estimated from the available data for a center of gravity located at the 0.252 $\bar{C}$ (wing mean aerodynamic chord) station and an airplane in the landing configuration. Although the variations of C_L, C_D and C_M with angle of attack, α, were estimated for zero elevator deflection, the use of the linear stability derivatives such as $C_{L_{\delta_e}}$ and $C_{M_{\delta_e}}$ allowed trimming the aircraft to any initial flight conditions. The C_L vs. α curve for $\delta_e = 0$ was selected for a trim $C_{L_{max}}$ of about 1.46 with the stall break being very gradual, which is indicative of a progressive flow separation starting at the wing trailing edge.

Aircraft reference data and physical characteristics are listed in Table I.

The choice of aircraft flight traits are arbitrary and may seem restrictive to a person who is solely concerned about a particular aircraft by virtue of personal experiences. However, it should be recognized that the methods and qualities described in this analysis are typical and may be modified to a particular application by changing the input data.

C. FLIGHT PROCEDURES

All "test flights" were conducted with the F-94A configurated for a landing approach. The aircraft was trimmed for steady descending flight at 130 percent of the estimated minimum flying speed. Aircraft thrust, weight and moment of inertia about the lateral axis were the only physical characteristics changed during the "test flights".

TABLE I

a. Physical Data

Parameter	Value
S	239 ft^2
$\bar{c}$	6.4 ft
W	12,359 lb
I_y	26,545 slug-ft^2
C_{L_α} (linear region)	5.27 rad^{-1}
$C_{L_{\delta_e}}$	0.43 rad^{-1}
C_{M_α} (linear region)	-0.406 rad^{-1}
$C_{M_{\delta_e}}$	-0.88 rad^{-1}
$C_{M_{\dot{\alpha}}}$	-4.268 rad^{-1}
C_{M_q}	-8.165 rad^{-1}
C	25.2% $\bar{c}$

TABLE I (Cont'd)

b. Tabular Data (complete aircraft, $\delta_e = 0$)

α (rad)	C_L	C_M	C_D
0.10472	0.552	-0.0027	0.0940
0.13963	0.738	-0.0170	0.1150
0.17453	0.921	-0.0315	0.1400
0.20944	1.093	-0.0445	0.1695
0.24435	1.233	-0.0585	0.2030
0.27925	1.346	-0.0740	0.2490
0.31416	1.430	-0.0910	0.3120
0.34907	1.484	-0.1120	0.3640
0.38397	1.513	-0.1320	0.4050
0.41888	1.520	-0.1520	0.4550
0.45379	1.500	-0.1720	0.4930
0.48869	1.462	-0.2075	0.5350
0.52360	1.415	-0.2400	0.5720
0.55851	1.360	-0.2795	0.6100

Deceleration from the trim condition was accomplished by decreasing the elevator angle, to simulate aft control stick movement. Available time histories, [Ref. 4], indicated elevator angle to be a near linear function of real-time during the deceleration maneuver. Therefore, a constant rate of change of elevator angle was chosen for this study. Rate of change of elevator angle was varied between -0.05 and -1.15 deg/sec. for the different maneuvers.

The effects of thrust were studied by making two "test flights" for each elevator angle schedule. Two thrust values were used for the "flights" which gave thrust to weight ratios of 0.02 and 0.11.

Five "flights" were made at -0.286 deg/sec. rate of change of elevator angle while varying aircraft weight to study the effects of aircraft weight on minimum flying speed. The aircraft weight was varied from 12,359 pounds to 16,359 pounds in increments of 1000 pounds.

"Flights" were made at representative rates of change of elevator angle to study the effects of aircraft moment of inertia about the lateral axis, on minimum flying speed. Values 25 percent above and below the true aircraft moment of inertia were evaluated.

Data from the original "test flight" indicated the possible exist-ance of an optimum elevator schedule in determining minimum flying speed. An exponential elevator schedule was studied and compared to the linear elevator schedule. The aircraft was decelerated using a linear schedule until the deceleration rate fell below a preselected rate. At this point the elevator schedule was changed to an exponential schedule until minimum flying speed was obtained.

D. DATA-ANALYSIS METHODS

Five data-analysis methods were used to determine minmum flying speed from the two main sets of "flight test" data. These methods were:

1. Constant Deceleration Rate

The linear elevator angle schedule resulted in near constant deceleration rates by the test aircraft. Minimum flying speed was defined as the actual minimum speed obtained during the deceleration. For consistency, a deceleration rate was defined as the slope of a straight line drawn from V_{min} to T.T. V_{min}. This deceleration rate was used in both the C_L and C_L' methods. Plots were made of minimum flying speed as a function of deceleration rate and minimum flying speed as a function of rate of change of elevator angle for the two values of thrust.

The constant deceleration rate method was used to analyze data when the effects of aircraft weight and moment of inertia on minimum flying speed were studied.

2. C_L' Methods

C_L' was defined as the aircraft left coefficient independent of normal flight path acceleration:

$$C_L' = \frac{W}{\bar{q}S}$$

The assumption of aircraft weight and aircraft lift being equivalent throughout the maneuver corresponds to considering C_L' as being an apparent lift coefficient rather than a true value. Minimum flying speed was defined by the C_L' method as the speed of the aircraft when C_L' was at a maximum value. Plots were made of $C_{L_{max}}'$ as a function of

deceleration rate, $V_{C'_{max}}$ as a function of deceleration rate and $V_{C'_{min}}$ as a function of rate of change of elevator angle.

3. $\underline{C_L \text{ Method}}$

C_L was defined as being the true aircraft lift coefficient which compensated for effects of normal flight path acceleration:

$$C_L = \frac{W \cdot a_{n_{fp}}}{\bar{q}S}$$

Minimum flying speed was defined by the C_L method as the speed of the aircraft when C_L was at a maximum value. Plots were made of $C_{L_{max}}$ as a function of deceleration rate, $V_{C_{min}}$ as a function of deceleration rate and $V_{C_{min}}$ as a function of rate of change of elevator angle.

4. <u>Altitude Break Method</u>

The aircraft altitude trace had a relatively linear characteristics slope during the deceleration maneuver. With the onset of stall the characteristic slope would increase significantly in value. The minimum flying speed was defined as the speed of the aircraft where the linear characteristic slope of the altitude trace could no longer be maintained. The characteristic slope increase was indicative of increased altitude loss.

A deceleration rate was defined as the slope of a straight line drawn from $V_{h_{min}}$ to 1.1 $V_{h_{min}}$. Plots were made of $V_{h_{min}}$ as a function of rate of change of elevator angle.

5. <u>1-g Break Method</u>

The minimum flying speed defined by this method was the last speed at which the aircraft could maintain 1-g flight. This method proved unreliable in data reduction for an aircraft trimmed in steady,

descending flight and was thus not pursued further. For this method, an aircraft must be decelerated from an initial climb condition.

III. RESULTS AND DISCUSSION

A. ABSOLUTE MINIMUM FLYING SPEED

1. Constant Deceleration Rate Method

The minimum flying speeds determined were plotted in Figure 3 as a function of deceleration for thrust-to-weight ratios 0.02 and 0.11. For a thrust-to-weight ratio of 0.02 the absolute minimum flying speed was 83.7 knots and occurred at a deceleration rate of -2.60 knots/sec. A thrust-to-weight ratio of 0.11 decreased the absolute minimum flying speed to 80.6 knots with no appreciable change in deceleration rate. Figure 2 also indicated absolute minimum flying speed was sensitive to variation in optimum deceleration rate. The addition of thrust reduced this sensitivity somewhat.

Figure 4 indicated absolute minimum flying speed was relatively insensitive to rate of change of elevator. Rates between -0.7 and -1.1 deg/sec. were able to give a close approximation of absolute minimum flying speed.

2. C_L' Method

Maximum values of C_L' were plotted in Figure 5 as a function of deceleration rate. $V_{C'_{min}}$ was plotted as a function of deceleration rate in Figure 6 and as a function of rate of change of elevator angle in Figure 7. A C_L' of 1.815 was the maximum attainable in steady, unaccelerated flight. The absolute minimum flying speed was 83.5 knots. The thrust-to-weight ratio of 0.11 decreased the absolute minimum flying speed three knots with minimal effects on deceleration rate. $V_{C'_{min}}$ was fairly sensitive to deceleration rate and insensitive to rate of change of elevator angle.

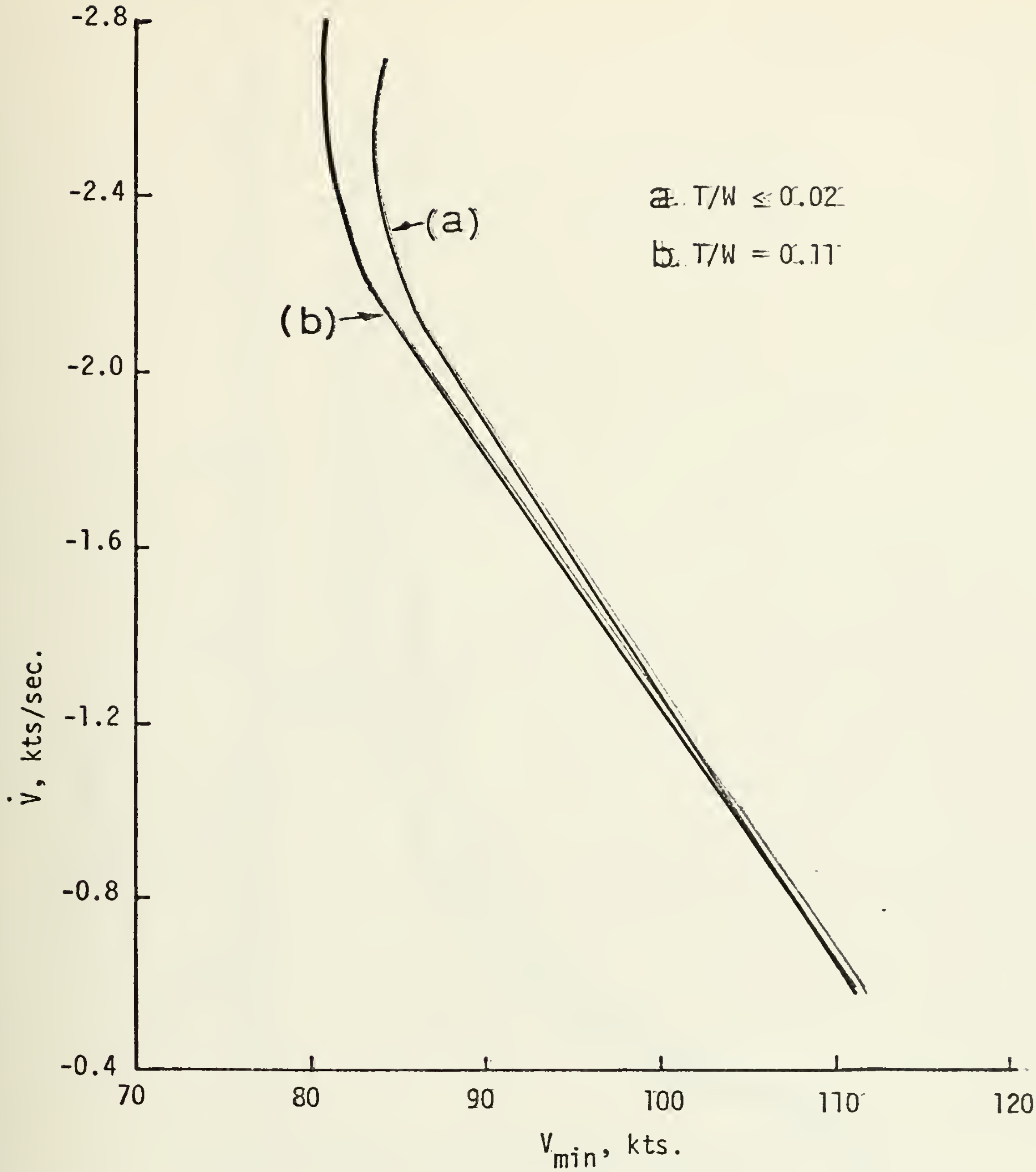

Figure 3. Minimum Flying Speed as a Function of Deceleration Rate.

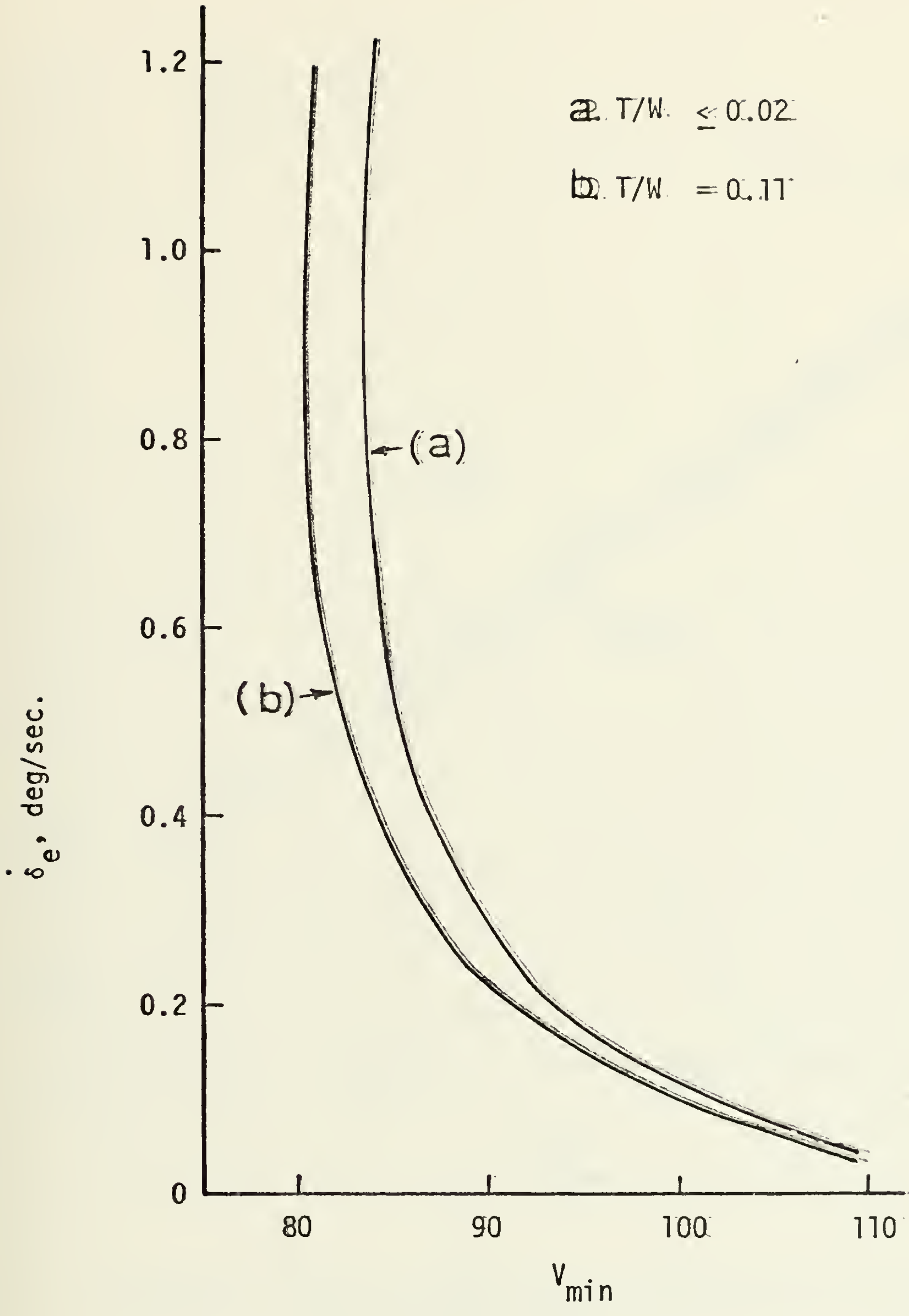

Figure 4. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

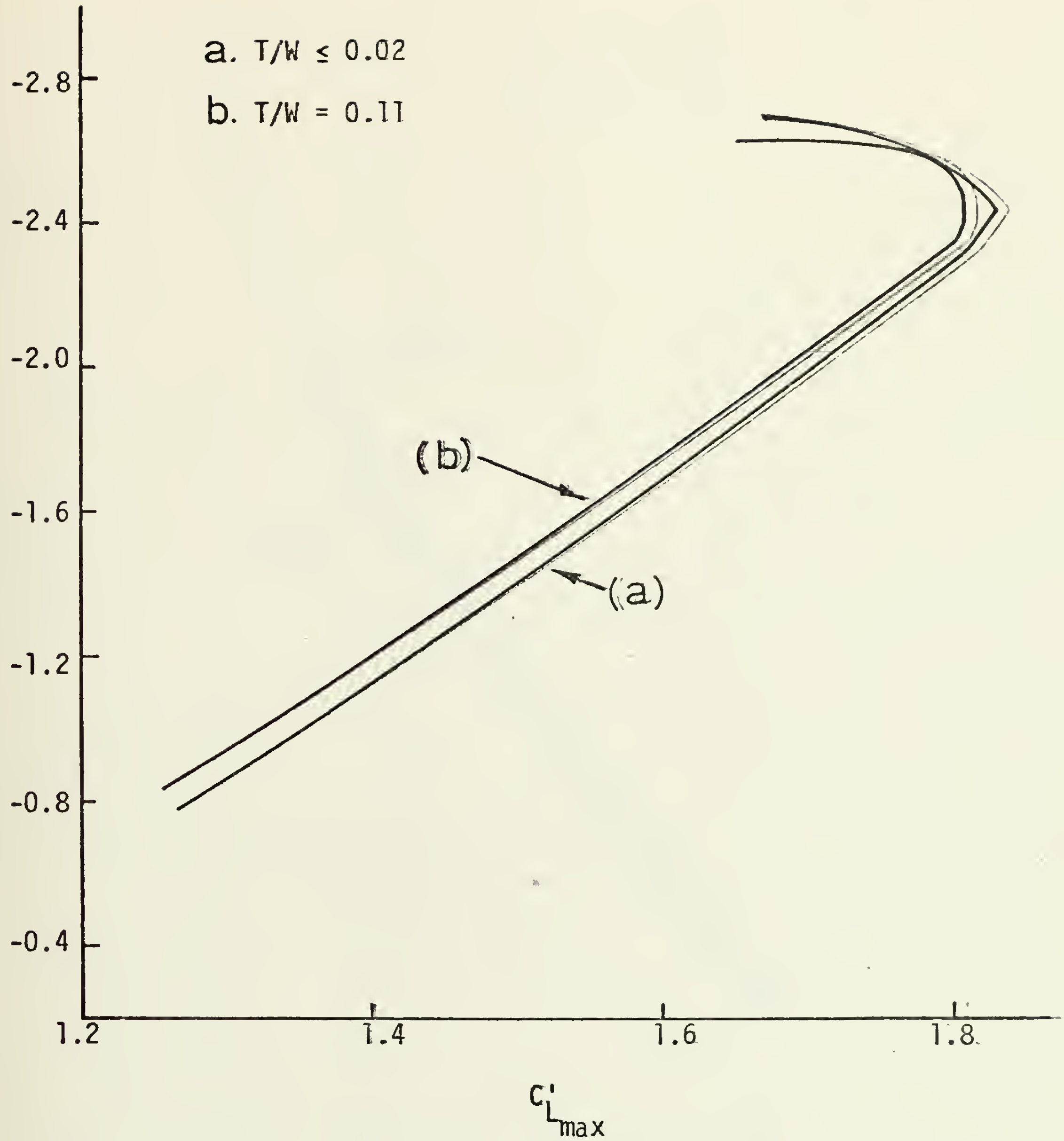

Figure 5. Apparent Maximum Lift Coefficient as a Function of Deceleration Rate.

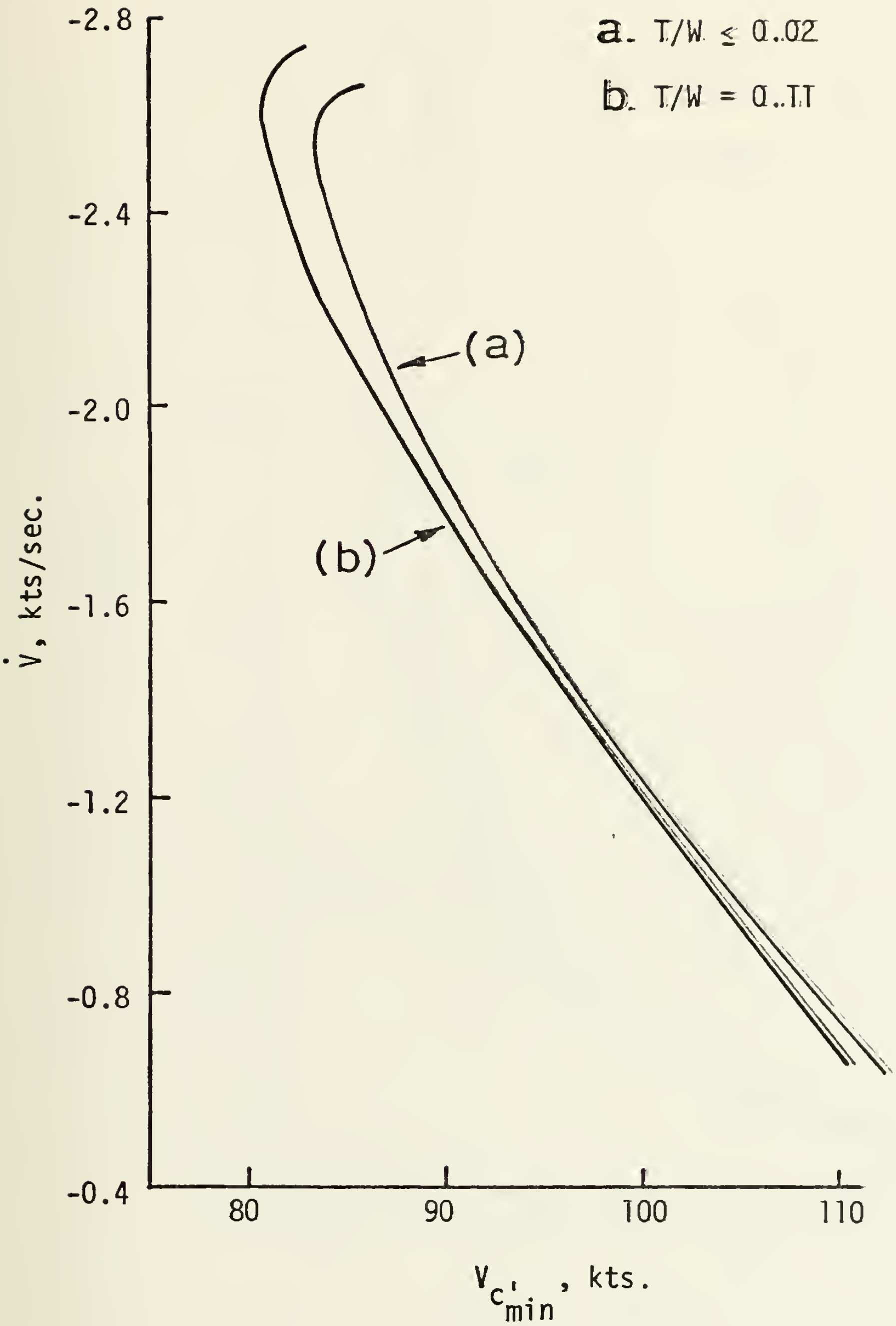

Figure 6. Minimum Flying Speed as a Function of Deceleration Rate.

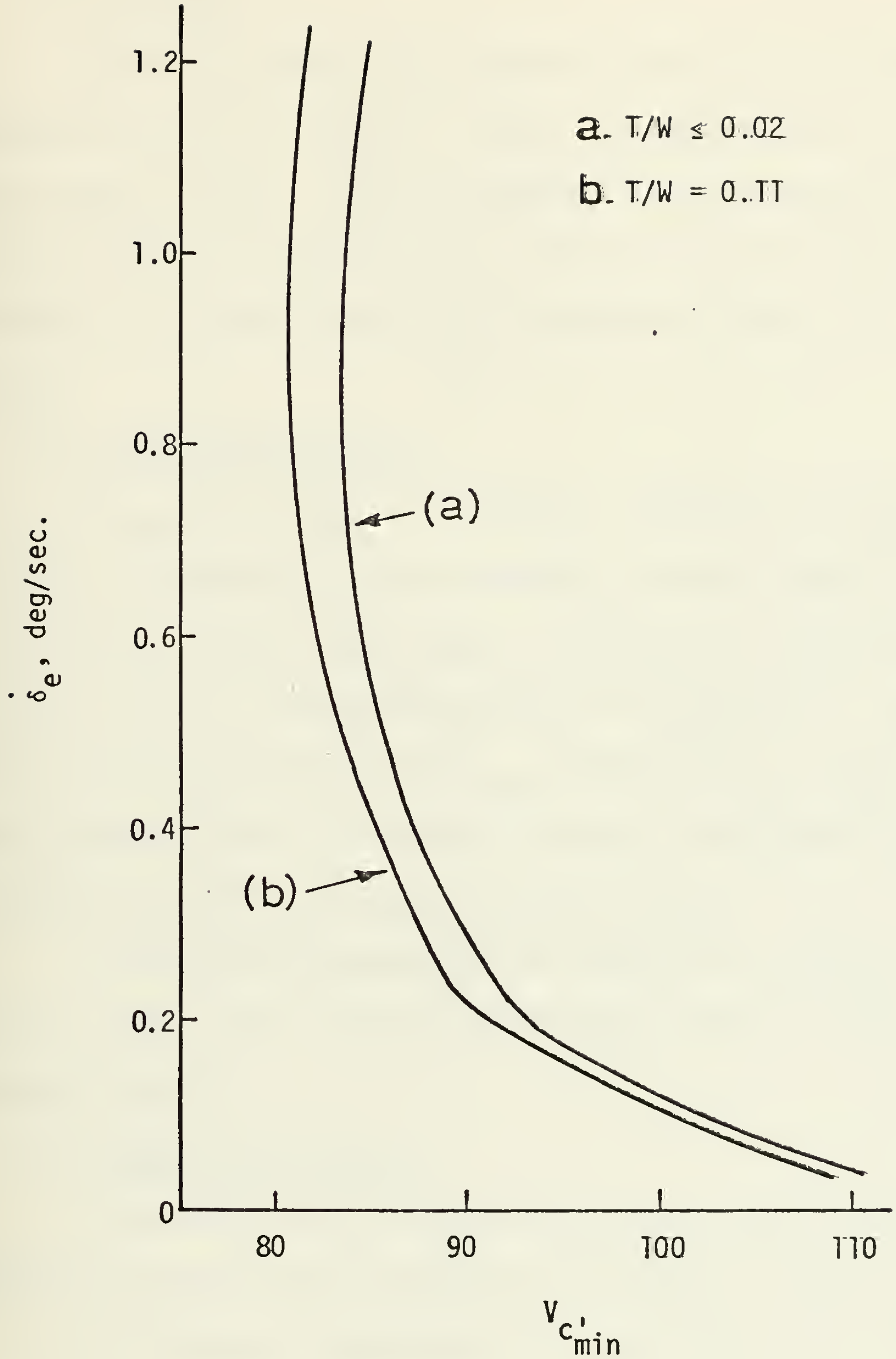

Figure 7. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

3. C_L Method

Absolute minimum flying speed was 88 knots for a thrust-to-weight ratio of 0.02, Figures 8 and 9. Increasing thrust decreased this speed 2.5 knots. $V_{C_{min}}$ was sensitive to both deceleration rate and rate of change of elevator angle. Figure 10 indicated the absolute maximum C_L value was insensitive to deceleration rate and increased thrust.

4. Altitude Break Method

$V_{h_{min}}$ was plotted as a function of deceleration rate in Figure 11 and as a function of rate of change of elevator angle in Figure 12. The absolute minimum flying speed for a thrust-to-weight ratio of 0.02 was 83.5 knots at a deceleration rate of -2.6 knots/sec. Figure 11 indicated that two values for $V_{h_{min}}$ existed for one value of deceleration rate between -2.6 and -3.76 kts/sec. Figure 12 indicated $V_{h_{min}}$ was insensitive to rate of change of elevator angle.

The altitude break method was dependent on graphical interpretation for data. This accounts for the small amount of data scatter in Figures 11 and 12.

5. Comparison of Methods

The general relationships of absolute minimum flying speed as determined by the various methods are indicated in Figure 13.

The constant deceleration, C_L and altitude break methods defined the absolute minimum flying speed to be 83 knots, Figure 13. The C_L method absolute minimum flying speed was 88 knots. Increased thrust decreased the absolute minimum flying speed defined by the four methods about three knots.

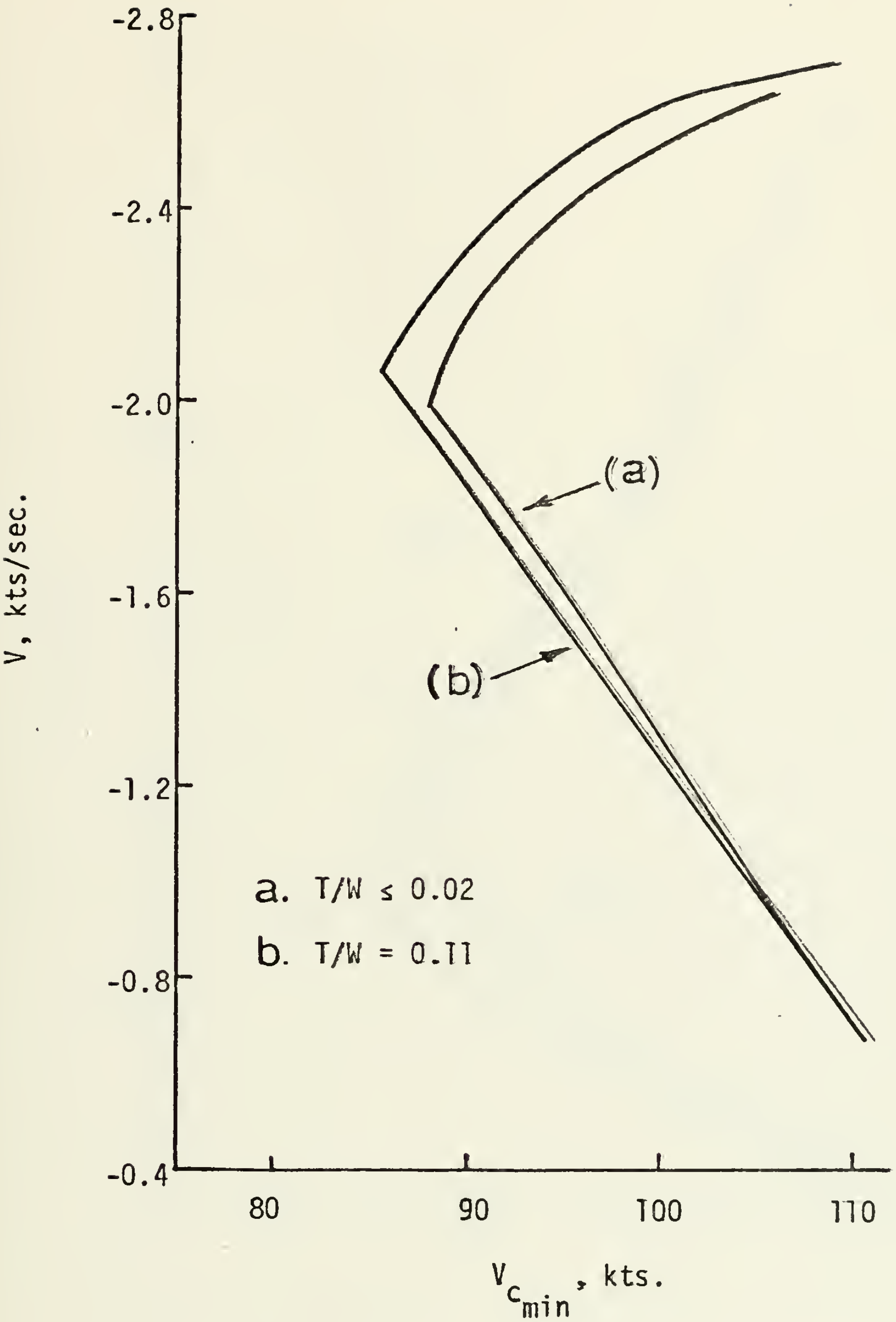

Figure 8. Minimum Flying Speed as a Function of Deceleration Rate.

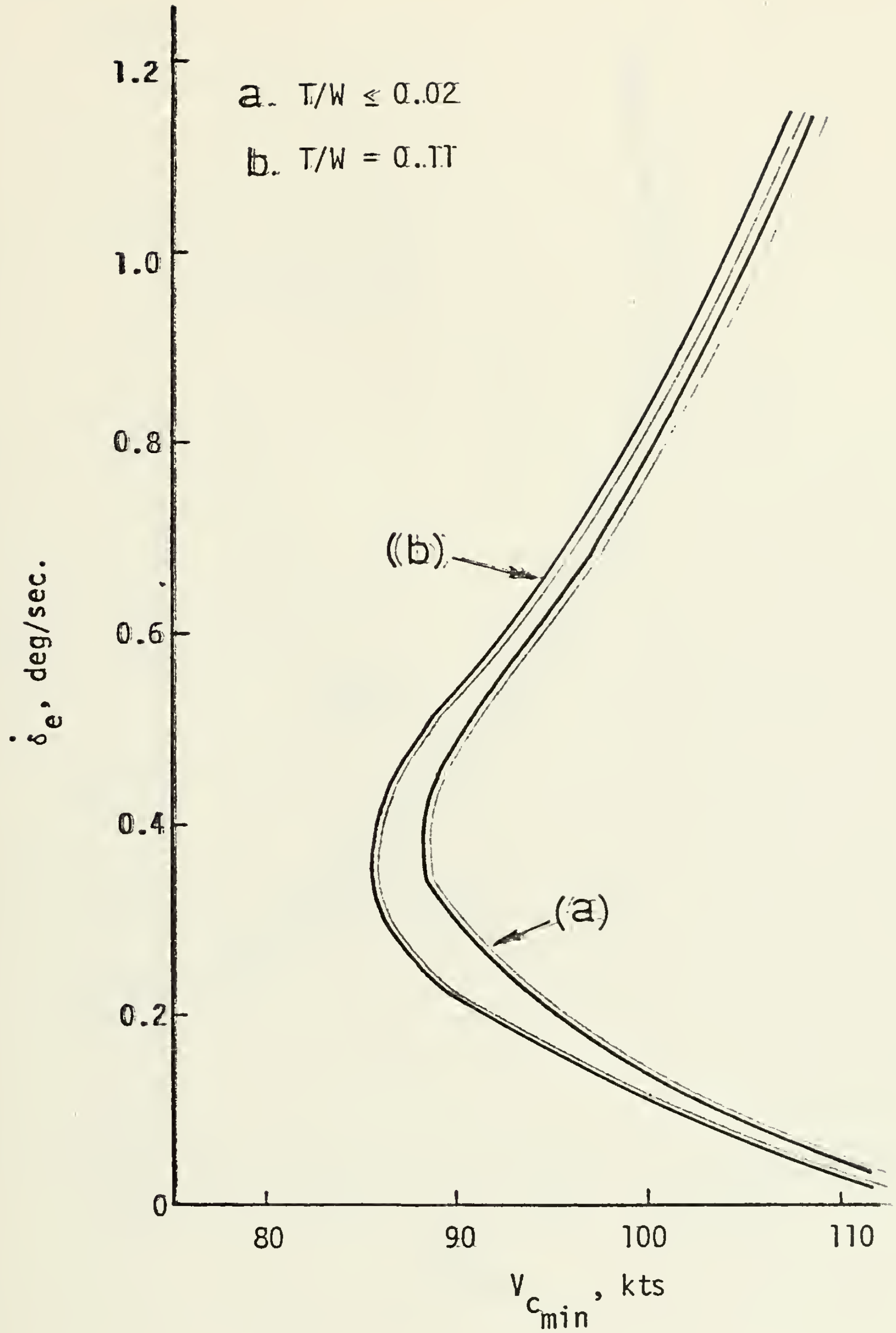

Figure 9. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

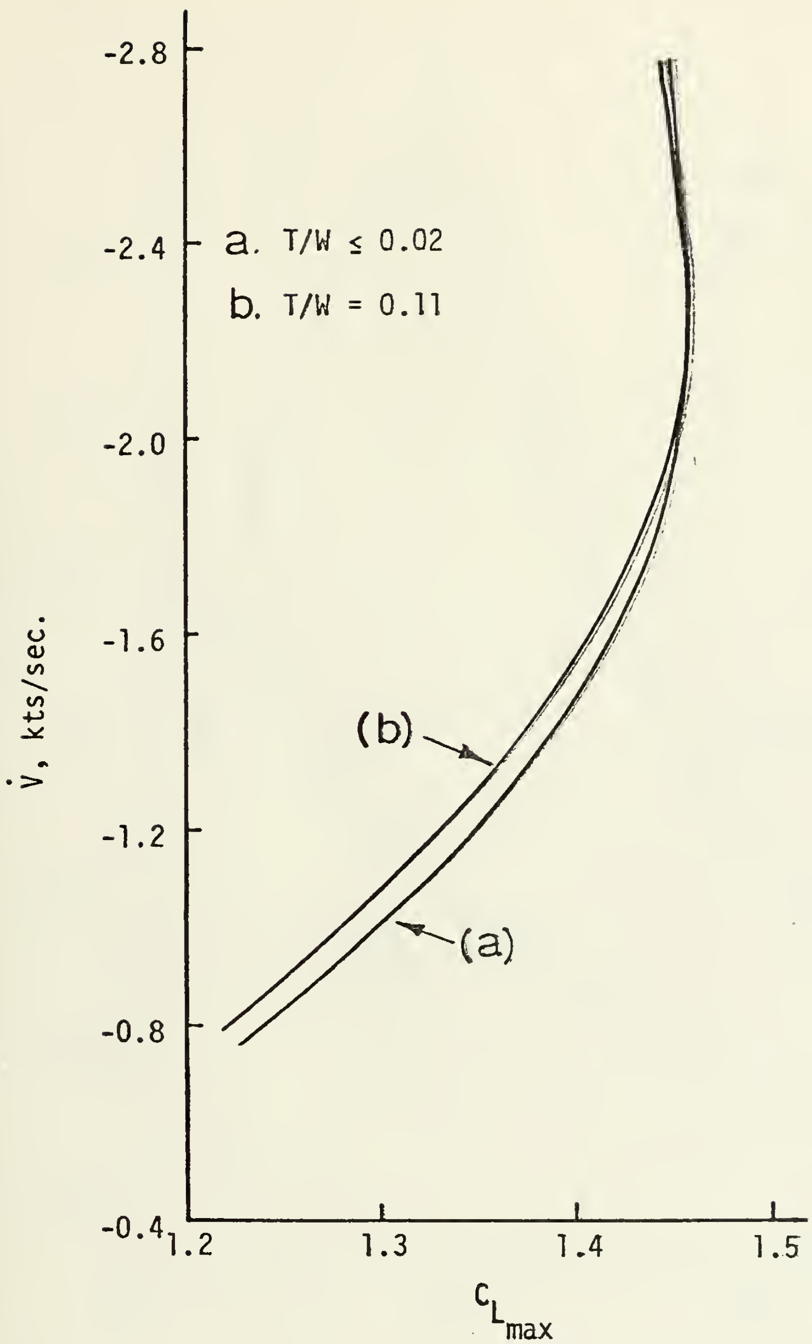

Figure 10. True Maximum Lift Coefficient as a Function of Deceleration Rate.

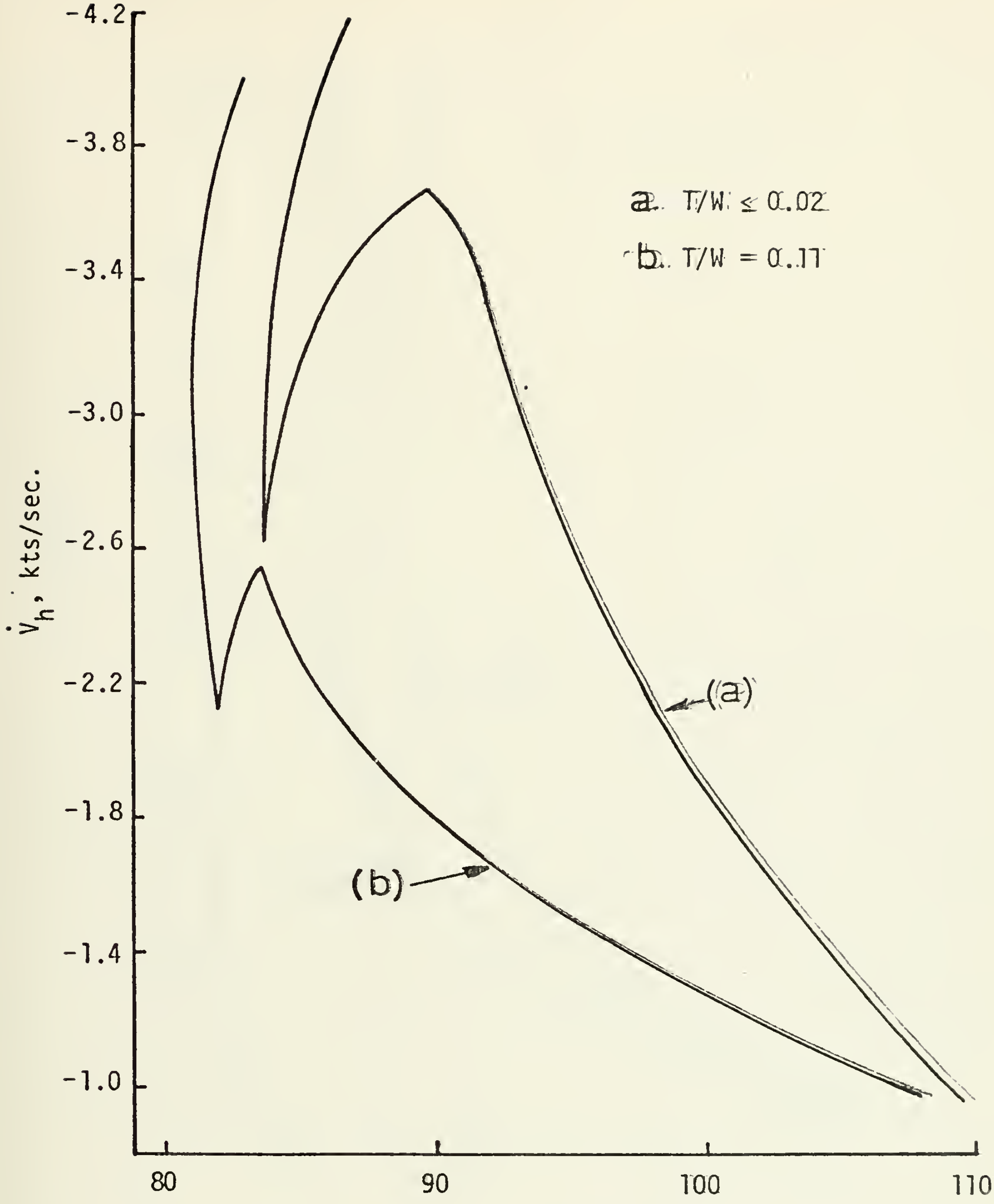

Figure 11. Minimum Flying Speed as a Function of Deceleration Rate.

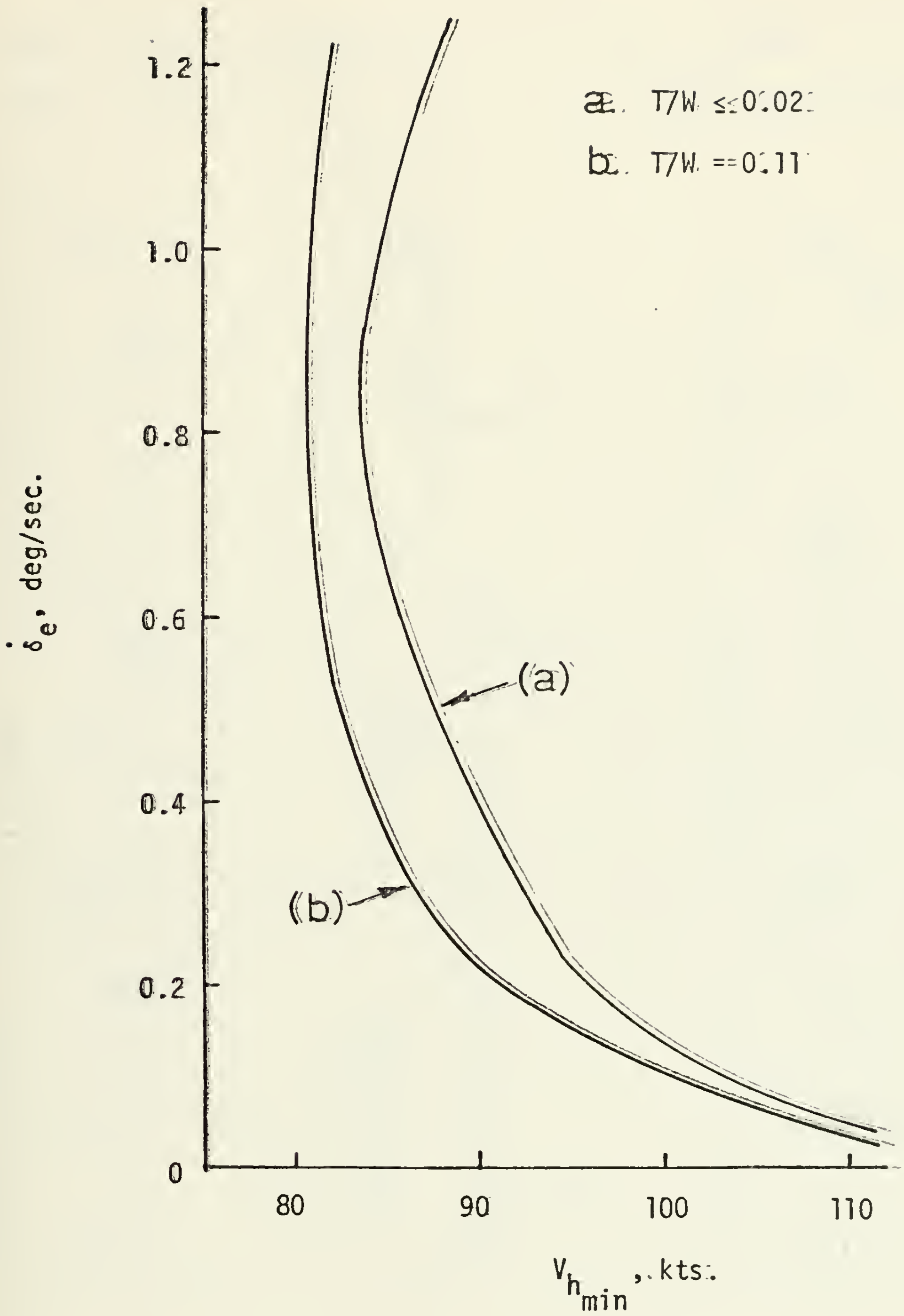

Figure 12. Minimum Flying Speed as a Function of Rate of Change of Elevator Angle.

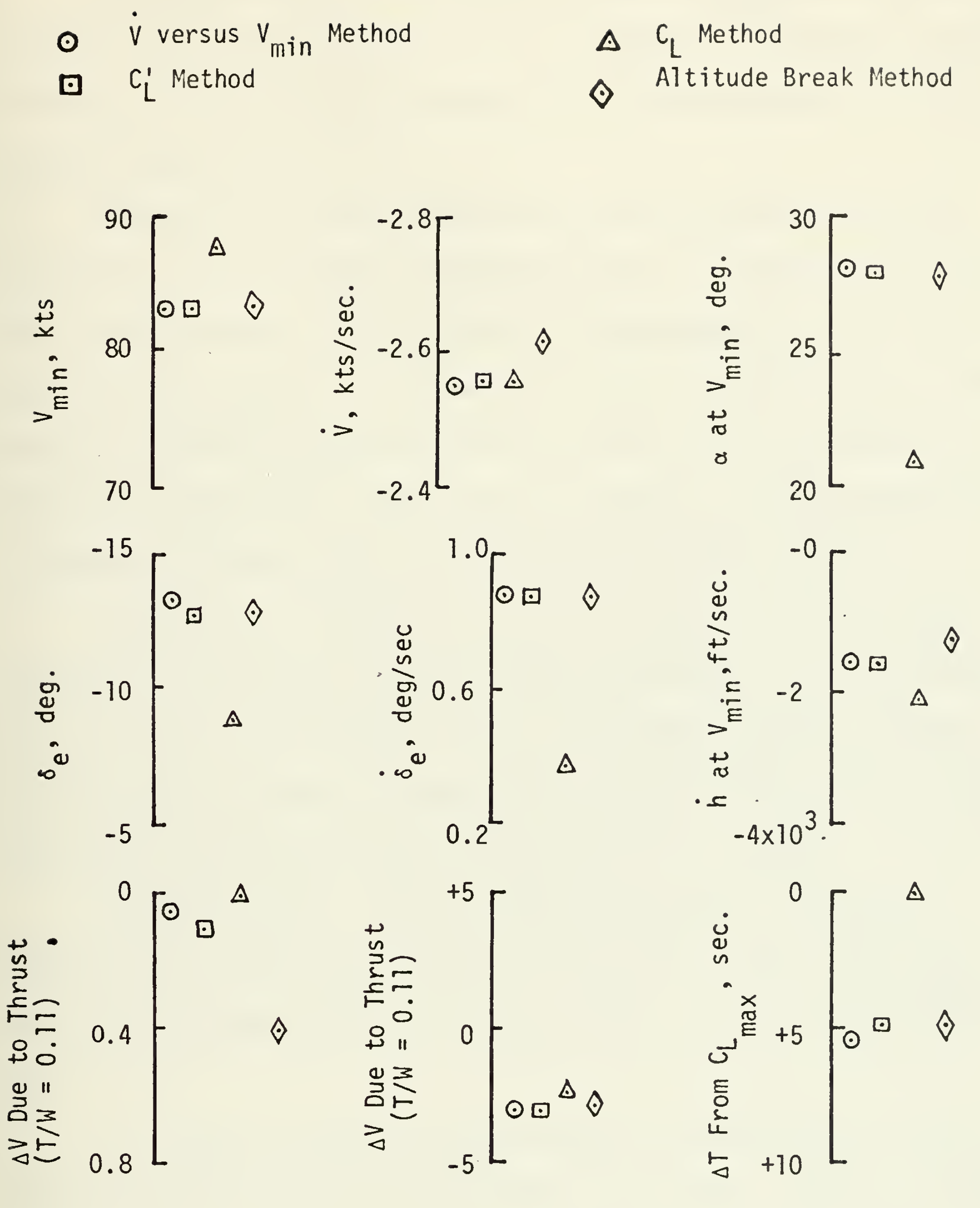

Figure 13. Comparison of Aircraft Parameters at Absolute Minimum Flying Speed T/W ≤ 0.02.

The deceleration rate was approximately -2.6 knots/sec at the absolute minimum flying speed for all four methods. Significant changes in deceleration rate due to increased thrust was observed only in the altitude break method.

Minimum flying speed determined by the $\dot{C}_L$ method occurred at the smallest rate of change of elevator angle, -0.4 degs/sec. Minimum flying speed determined by the constant deceleration, C_L and altitude break methods occurred at -0.9 deg/sec. rate of change of elevator angle. Absolute minimum flying speed defined by the $\dot{C}_L$ method occurred at the smallest elevator angle, approximately -9 degrees, whereas approximately -13 degrees elevator angle was required for the other methods.

The angle of attack at the absolute minimum flying speed was approximately 28 degrees for the constant deceleration, C_L and altitude break methods, and considerably less at 21 degrees for the $\dot{C}_L$ method. Thrust effects on angle of attack at the absolute minimum flying speed were insignificant (less than 0.5 degrees) for all methods.

Elevator angles, lift coefficients and angle of attacks at minimum flying speed/stall speed were less than, in some cases, the maximum static values defined for the aircraft. Possible explanations are:

1. Rate of change of elevator angle was programmed to become zero two seconds after V_{min} was obtained. For small values of elevator rate, V_{min} occurred much sooner than the maximum obtained values of C_L, α and δ_e. Therefore, elevator angle was held constant until the end of the maneuver, restricting the aircraft from obtaining the maximum values of C_L, α and δ_e.

2. The variations of drag coefficient with angle of attack used in the table look up were representative values and not necessarily true values for the F-94A. Reduced drag coefficients would therefore effect the occurrance of minimum flying speed.

3. In defense of the low values of C_L, α and δ_e, it must be remembered that this is a dynamic study of stall traits. Actual flight test data, [Ref. 4], indicates angle of attack is less than static stall angle of attack.

B. FAR MINIMUM FLYING SPEED

The minimum flying speed defined by Federal Aviation Regulations corresponds to the minimum speed obtained at a one knot/sec. deceleration rate. The FAR minimum flying speed determined by the constant deceleration, C_L and C_L' methods was approximately 105 knots. The altitude break method's FAR minimum flying speed was 109 knots. Increased thrust decreased the altitude break minimum speed two knots and decreased the minimum speed defined by the remaining methods less than one knot.

The rate of change of elevator was − 0.08 deg/sec. at minimum flying speed determined by the constant deceleration rate, C_L and C_L' methods. The altitude break method rate of change of elevator angle was -0.05 deg/sec.

Angle of attack was approximately 15 degrees and elevator angle -4.8 degrees for all methods.

C. COMPARISON OF MINIMUM FLYING SPEEDS

The absolute minimum flying speeds were categorically 20 knots less than those defined by FAR one knot/sec. deceleration rate, Figure 14. Thrust had more effect on the absolute minimum flying speed whereas rate

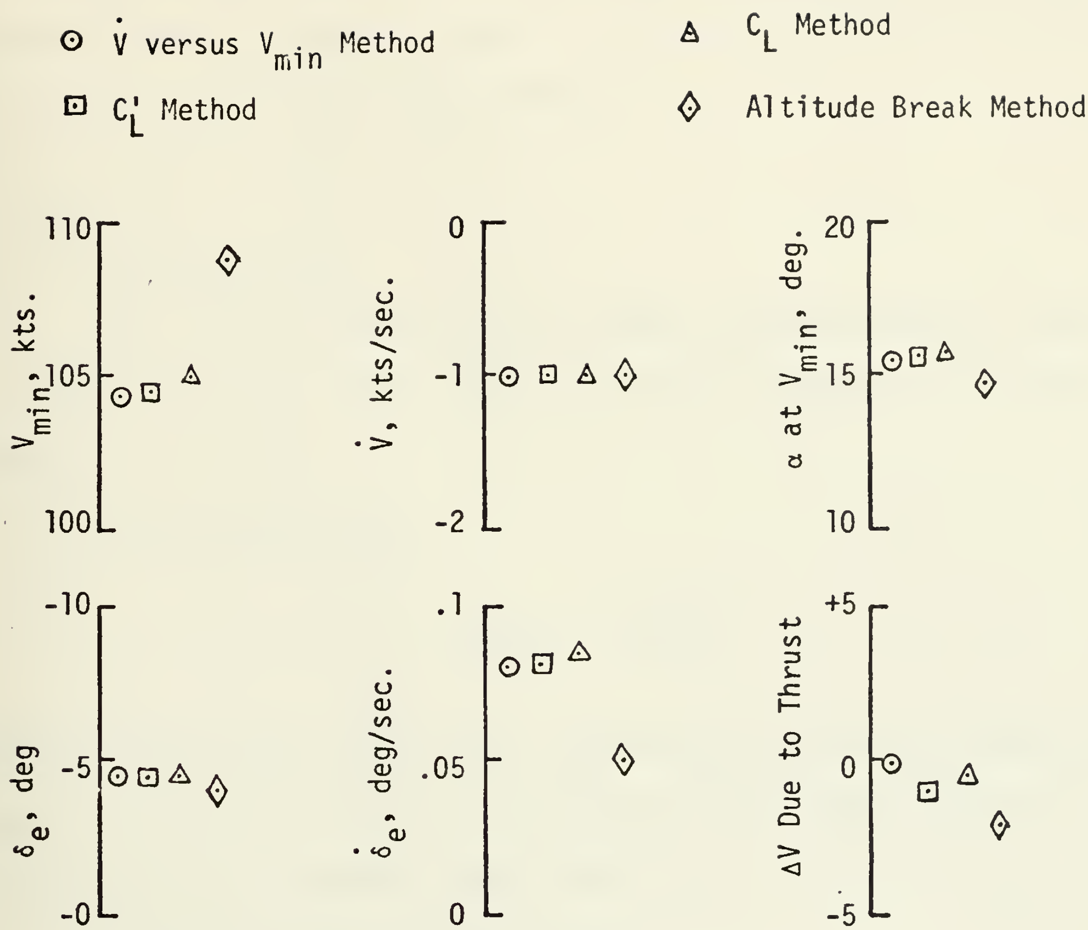

Figure 14. Comparison of Aircraft Parameters at FAR Minimum Flying Speed.

of change of elevator angle had more effect on the FAR minimum flying speeds. Angle of attack and elevator angle were significantly greater with the absolute minimum flying speeds.

D. SENSITIVITY STUDIES

1. Aircraft Weight

For a representative rate of change of elevator angle, Figure 15 indicates minimum flying speed was a linear function of aircraft weight.

From the definition:

$$C_{L_{max}} = \frac{W}{\overline{q}_s S}$$

minimum flying speed is seen to be a function of the one-half power of weight. Actual flight test data indicates minimum flying speed is a linear function of aircraft weight [Ref. 4].

2. Aircraft Pitch Moment of Inertia

When the aircraft pitch moment of inertia was increased or decreased 25 percent, there were no significant effects on minimum flying speeds or other aircraft parameters. This conclusion was not based on a systematic study but rather a brief look at three data runs. The trends were present but a complete study is warranted.

3. Exponential Elevator Rate

Comparative plots of linear and linear plus exponential elevator rates were made in Figure 16 for initial linear rates of -0.06 and -0.115 deg/sec. The exponential rate was e^x where x was increased 0.04 each second. V_{min} obtained by the linear plus exponential elevator rate was significantly less than that obtained by the linear elevator rate.

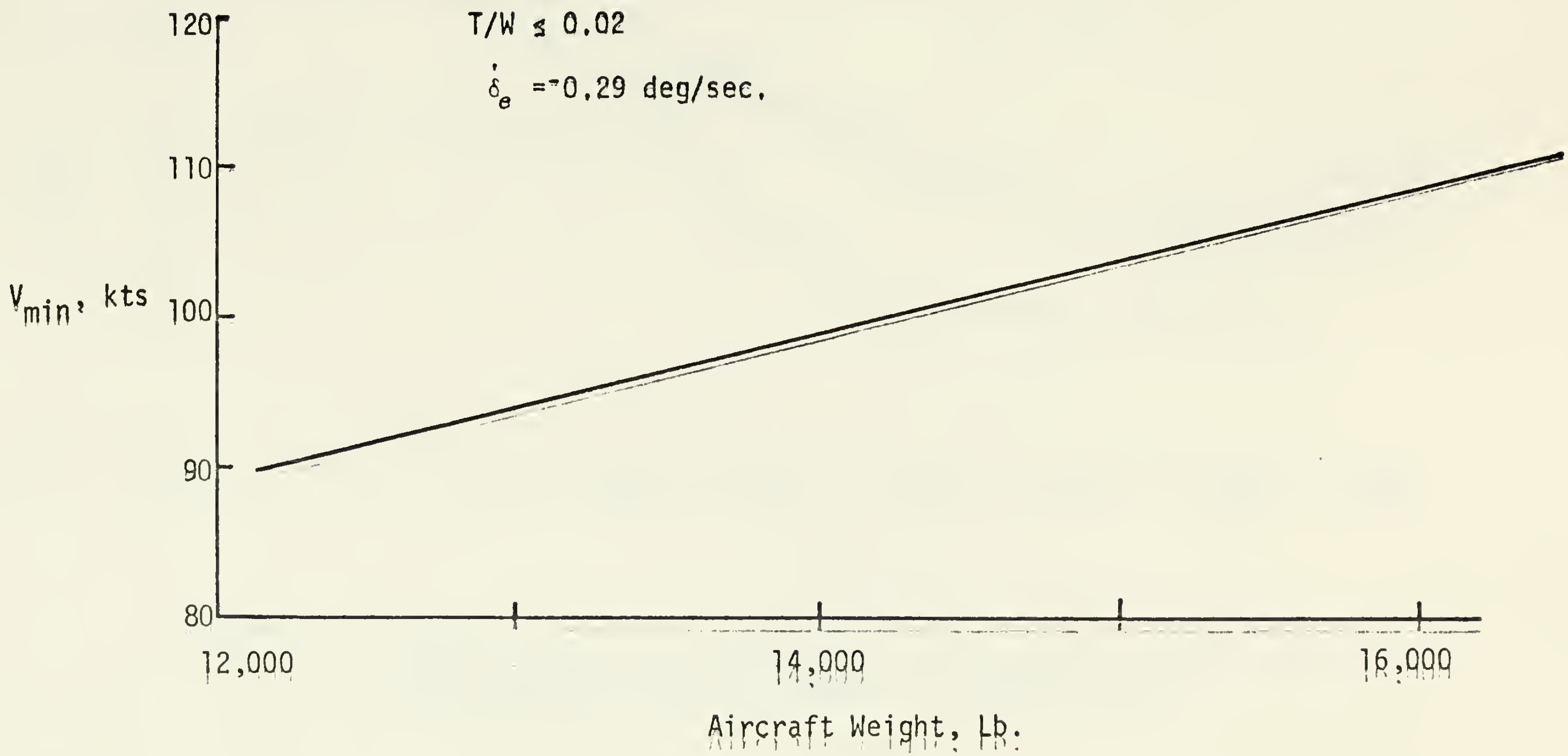

Figure 15. Minimum Flying Speed as a Function of Aircraft Weight.

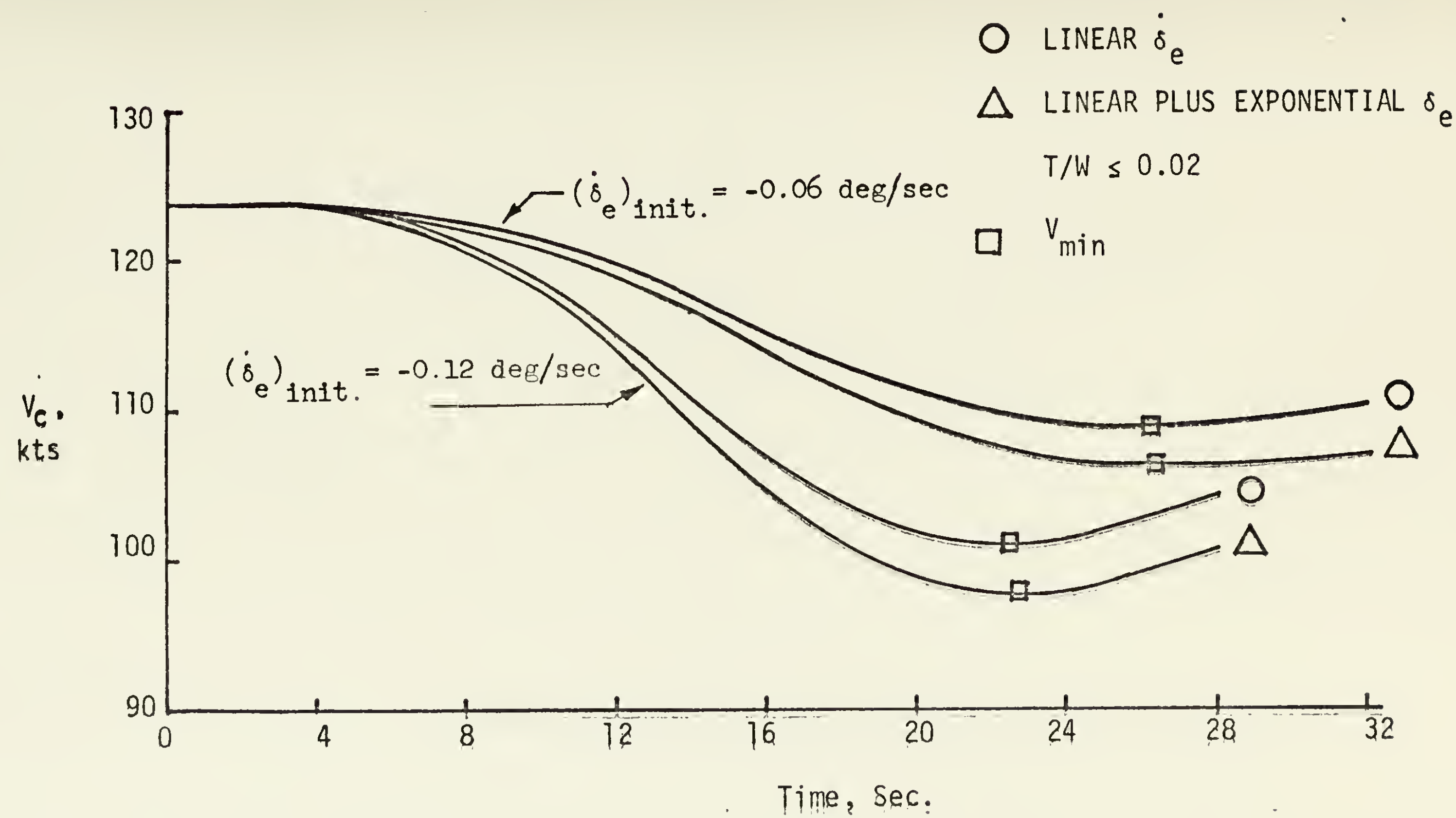

Figure 16. Comparison of Aircraft Velocity Time History for Linear and Linear Plus Exponential Rate of Change of Elevator Angle.

These results, though only a sampling of data, tend to support the existance of an optimum elevator schedule in determining minimum flying speed.

IV. CONCLUSIONS AND RECOMMENDATIONS

Digital simulation has great potential in the evaluation of minimum flying speed. The options available are practically unlimited.

The determination of absolute minimum flying speed and aircraft behavior with digital simulation will enable the engineer to better predict the aircraft characteristics encountered during test flight. The test pilot's job will be simplified somewhat if he has knowledge of general aircraft characteristics prior to flight.

The operational pilot's performance should be enhanced by having an insight into his aircraft's performance beyond the certified stall speed.

The computer program coded for this study may be used for many fields of study as indicated by Ref. 7, where it was used to make a stability analysis in the vicinity of minimum flying speed.

APPENDIX A

NON-LINEAR EQUATIONS OF AIRCRAFT MOTION

I. Euler's Equations of Aircraft Motion.

The aircraft was considered as a rigid body with a Cartesian coordinate system fixed at the center of mass, Figure 17. The frame of reference Cxyz, a body axis system, moved with the aircraft and Cx was fixed to the longitudinal axis of the aircraft. The resultant external forces and the moments of these forces about the aircraft center of mass, when referred to the frame of reference Cxyz, where the vector equations of motion:

$$\overline{F} = m \frac{\partial \overline{V_c}}{\partial t} + m \, \overline{\omega} \times \overline{V_c}$$

$$\overline{G} = \frac{\partial \overline{h}}{\partial t} + \overline{\omega} \times \overline{h}$$

The scalar components of these equations are Euler's equations of aircraft motion.

II. Aircraft Orientation and Axes.

Reference frame Cxyz could not be used to describe the position and orientation of the aircraft as it was fixed to the center of mass and moved with the aircraft. An earth fixed reference frame, assumming the earth's rotation was negligible, was chosen to describe the aircraft motion.

Euler's equations of motion are valid for any orthogonal reference frame fixed to the aircraft center of mass. To simplify the equations of motion and expressions for aerodynamic forces, the reference frame,

Cx'y'z', was chosen as stability axes, i.e., Cx' pointed in the direction of motion of the aircraft center of mass, Figure 17.

III. Assumptions

In addition to the assumptions previously made defining aircraft orientation and axes it was assumed that aircraft motion was restricted to the Cxy, longitudinal plane. This assumption equated the following quantities to zero: L, N, P, R, Φ, Ψ, V and Y. Cxy was also assumed to be a plane of symmetry which equated the products of inertia, D and F, to zero.

IV. Non-Dimensionalization

The external aerodynamic forces and moments were expressed as functions of dimensionless force and moment coefficients, dynamic pressure, aircraft planform wing area and a reference length. These equations, in scalar form, were:

$$X = C_x \frac{1}{2} \rho V_c^2 S$$

$$Z = C_z \frac{1}{2} \rho V_c^2 S$$

$$M = C_m \frac{1}{2} \rho V_c^2 S \bar{c}$$

When these values were substituted into the scalar resultant external force and moment equations the following equations were obtained:

$$\dot{U} = \frac{\rho V_c^2 S C_x}{2m} - g \sin \theta - QW$$

$$\dot{W} = \frac{\rho V_c^2 S C_z}{2m} + g \cos \theta + QU$$

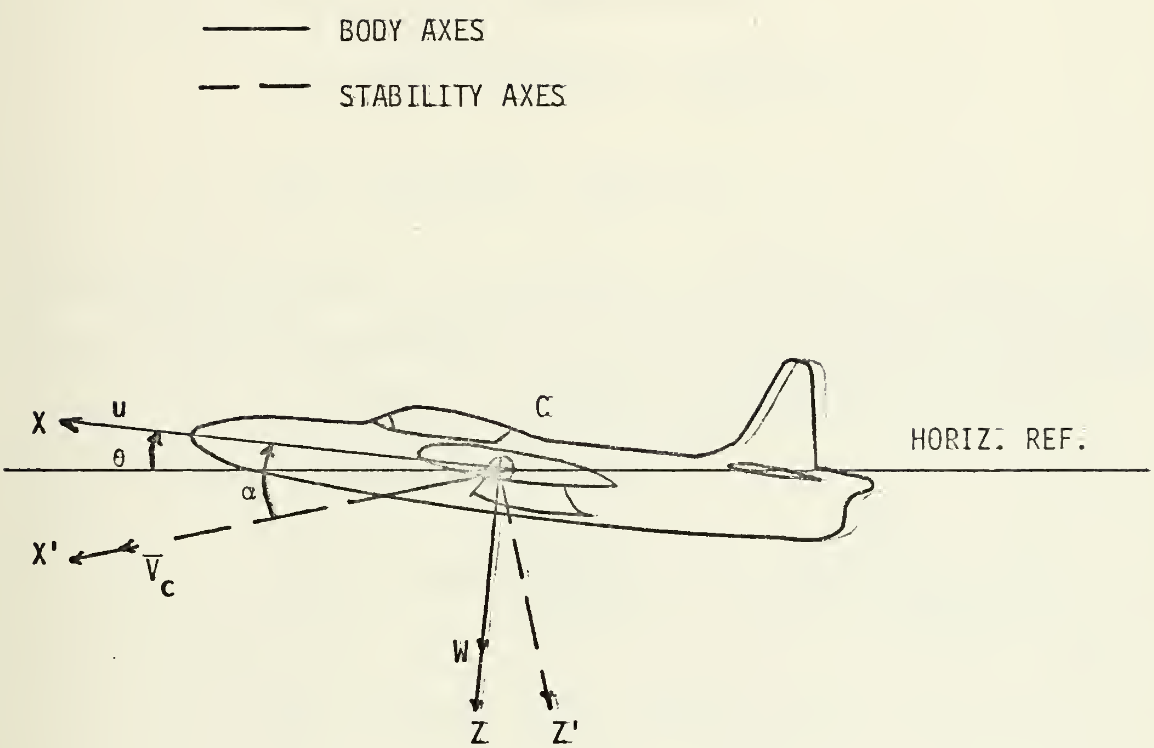

Figure 17. Aircraft Axes.

$$\dot{Q} = \frac{\rho V_c^2 S \bar{c} C_m}{B}$$

$$Q = \dot{\theta}$$

V. Aerodynamic Force and Moment Coefficients

From Figure 18, the force coefficients were:

$$C_x = C_T + C_L \sin \alpha - C_D \cos \alpha$$

$$C_z = -C_L \cos \alpha - C_D \sin \alpha$$

The pitching moment and lift coefficients consisted of linear portions. The non-linearity was present in $C_L(\alpha)$ and $C_m(\alpha)$, which were non-linear curves of C_L and C_m versus . Complete expressions for the total pitching moment and lift coefficients were:

$$C_m = C_m(\alpha) + \frac{c}{2V_c}\left[C_{m_q}\dot{\theta} + C_{m_{\dot{\alpha}}}\dot{\alpha}\right] + C_{m_{\delta_e}}\delta_e$$

$$C_L = C_L(\alpha) + C_{L_{\dot{\theta}}}\dot{\theta} + C_{L_{\delta e}}\delta_e$$

VI. Final Equations

Angle of attack appeared in the pitching moment equation as a derivative. To obtain a set of simultaneous differential equations that could be solved by one of the methods available, another equation was added defining the rate of change of angle of attack. The three-degrees-of-freedom, non-linear differential equations of aircraft motion programmed for the Naval Postgraduate School IBM 360 digital computer were:

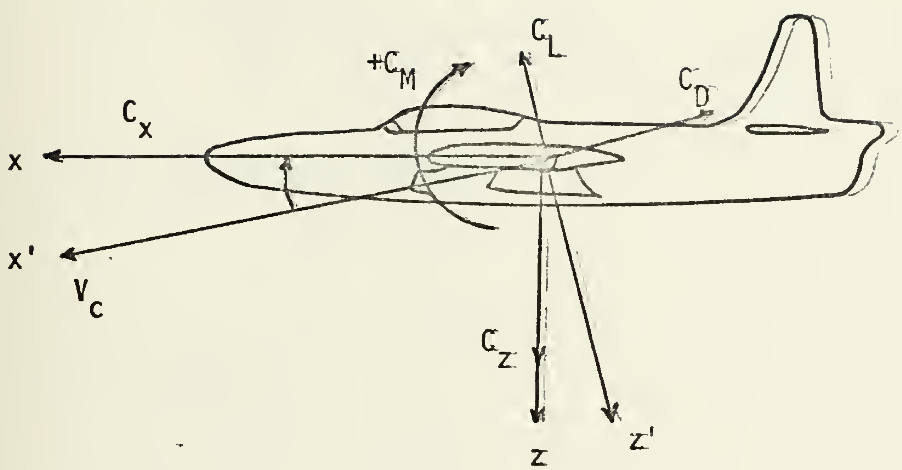

Figure 18. Aerodynamic Force and Moment Coefficients.

$$\dot{U} = \frac{\rho V_c^2 S}{2m} \left[C_T + \left(C_L(\alpha) + C_{L_{\dot{\theta}}} \dot{\theta} + C_{L_{\delta_e}} \delta_e \right) \sin \alpha - C_D \cos \alpha \right]$$

$$- g \sin \theta - QW$$

$$\dot{W} = \frac{\rho V_c^2 S}{2m} \left\{ - \left[\left(C_L(\alpha) + C_{L_{\dot{\theta}}} \dot{\theta} + C_{L_{\delta_e}} \delta_e \right) \cos \alpha + C_D \sin \alpha \right] \right\}$$

$$+ g \cos \theta + QU$$

$$\dot{\theta} = Q$$

$$\dot{Q} = \frac{\rho V_c^2 S \bar{c}}{2B} \left[C_m(\alpha) + \frac{c}{2V} \left(C_{m_{\dot{\alpha}}} \dot{\alpha} + C_{m_q} \dot{\theta} \right) + C_{m_{\delta_e}} \delta_e \right]$$

$$\dot{\alpha} = \frac{\dot{W} \cos \alpha - \dot{U} \sin \alpha}{U}$$

APPENDIX B

DEVELOPMENT OF COMPUTER PROGRAM

I. COMPUTER FACILITY

The original intent was to program the equations of motion on the Aeronautical Engineering Department EAI 580 analog computer. Problems with insufficient equipment to program the equations of motion on the analog computer necessitated the use of digital simulation.

The Naval Postgraduate School's IBM 360 digital computer offered many advantages and was chosen as the research tool. Several major advantages were: the large amount of core available, dimensioned quantities could be used, plot subroutines were available for time history studies and numerous methods were available for solving simultaneous non-linear differential equations on a digital computer.

II. COMPUTER PROGRAM

Of the many one step procedures that were available for solving systems of simultaneous first-order ordinary differential equations, a fourth-order Runge-Kutta algorithm was chosen, [Ref. 8]. This method offered simplicity, accuracy and short computer time. The Runge-Kutta procedures were programmed into subroutine RUNKUT. RUNKUT called function subroutines F1, F2, F3, F4 and F5 to calculate the derivative terms at time T(I), T(I)+HI/2 and T(I)+HI. The derivative terms were multiplied by the appropriate weight factors and added to the variables calculated at time T(I) which gave values for the variables at time T(I+1).

Function subroutine FT, called subroutine SPLIN to calculate the non-linearities in C_L, C_m and C_D. SPLIN was obtained from the scientific subroutine package of the Naval Postgraduate School IBM 360 digital computer and modified for this program. Non-linear tables of C_L, C_m and C_D as a function of angle of attack were read into SPLIN. When supplied values of angle of attack, SPLIN used a cubic spline function to provide interpolated values of C_L, C_m and C_D.

Initial aircraft trim conditions U, W, θ, $\dot{\theta}$, α and δ_e were supplied to RUNKUT at time zero for initiation of the step procedure. These values were calculated in Subroutine TRIM from static force and moment equations.

Control of the aircraft was accomplished through changes in elevator angle and thrust which simulated control stick and throttle movements. The options available were selected through the integers on the first data card.

III. TRUNCATION ERROR AND STEP-SIZE ANALYSIS

Error analysis by rigorous mathematical derivations are virtually impossible to implement for higher-order Runge-Kutta algorithms for systems of differential equations. The truncation error:

$$e_t = \frac{16}{15}\ (y_{n+1,2} - y_{n+1,1})$$

for a first-order ordinary differential equation was applied to the system of differential equations as a step-size control mechanism, [Ref. 8]. Step-size and truncation errors are listed in Table II.

Table II

Local Truncation Errors

Step-Size	Error
0.2	-2.92×10^{-4}
0.1	-3.97×10^{-5}
0.05	1.56×10^{-6}
0.025	-3.9×10^{-8}

```
C     PROGRAMED BY LT. C. W. SAUL TO OPERATE ON THE NAVAL
C     POSTGRADUATE IBM 360 DIGITAL COMPUTER.  PROGRAMED IN
C     CONJUNCTION WITH THE THESES,"EVALUATION OF MINIMUM
C     AIRCRAFT FLYING SPEED BY DIGITAL SIMULATION", FOR PARTIAL
C     FULFILLMENT OF THE REQUIREMENTS FOR A MASTERS DEGERE IN
C     AERONAUNTICAL ENGINEERING.
C
C
C     THE PROGRAM SOLVES THE NON-LINEAR, THREE-DEGREES-OF-FREEDOM
C     EQUATIONS OF AIRCRAFT MOTION BY USING A FOURTH-ORDER RUNGE-
C     KUTTA ALGORITHM.
C
C
C     THE PROGRAM WAS WRITTEN TO PROVIDE NUMEROUS OPTIONS IN THRUST
C     AND RATES OF CHANGE OF ELEVATOR ANGLE (CONTROLS THE AIRCRAFT
C     BY STICK AND THROTTLE MOVEMENTS).  THE OPTIONS WERE INCORPORATED
C     TO BE CONTROLED BY FOUR INTEGERS: I1,I2,I3 AND I4, LISTED ON
C     THE FIRST DATA CARD.  BY CHANGING ONLY THE FIRST DATA CARD,
C     THE VARIOUS OPTIONS IN THRUST AND RATE OF CHANGE OF ELEVATOR ANGLE
C     CAN BE CHOSEN.  VALUES FOR THE INTEGERS ARE PLACED IN THE FIRST
C     EIGHT COLUMNS OF THE DATA CARD WITH TWO SPACES ALLOWED FOR EACH
C     VALUE.  THE VALUES ARE RIGHT JUSTIFIED.  THE OPTIONS AVAILABLE
C     ARE:
C
C
C     I1        THRUST OPTION
C
C
C     VALUE                     OPTION
C
C
C       1       THRUST WILL BE CALCULATED FOR INPUT ANGLE OF ATTACK
C               AND PITCH ANGLE BY SUBROUTINE TRIM AND WILL REMAIN
C               CONSTANT THROUGHOUT THE FLIGHT
C
C       2       AIRCRAFT WILL BE TRIMMED AND FLOWN WITH ZERO THRUST
C
C       3       TRIM THRUST WILL BE THE SAME AS FOR I1=1 BUT WILL BE
C               GRADUALLY REDUCED TO A T/W=0.02 DURING THE FIRST
C               1.2 SECONDS OF FLIGHT
C
C       4       THE AIRCRAFT WILL BE TRIMMED AND FLOWN FOR T/W=0.02
C
C     I2        ELEVATOR SCHEDULE (I3 MUST EQUAL 1)
C
C
C     VALUE                      OPTION
```

```
C
C
C         1         DE DOT = -0.115 DEG/SEC
C
C         2         DE DOT = -0.229 DEG/SEC
C
C         3         DE DOT = -0.172    "
C
C         4         DE DOT = -0.286    "
C
C         5         DE DOT = 0         "
C
C         6         DE DOT = -0.115    "  UNTIL 2 SEC AFTER V MIN, THEN = 0
C
C         7         DE DOT = -0.229    "                  "
C
C         8         DE DOT = -0.172    "                  "
C
C         9         DE DOT = -0.286    "                  "
C
C
C        I3         FUTHER OPTIONS IN ELEVATOR SCHEDULE,
C
C
C        VALUE                          OPTION
C
C
C         1         HONORS THE ELEVATOR SCHEDULE CHOSEN BY I2
C         2,3,4     ALLOWS THE PROGRAMED TO SELECT A RATE OF
C                   DECELERATION FOR THE AIRCRAFT BUT WAS
C                   DESIGNED TO WORK IN CONJUNCTION WITH THE
C                   EXPONENTIAL ELEVATOR SCHEDULE DESCRIBED LATER.
C
C         5         THE ELEVATOR CHOSEN BY I2=6,7,8,9 IS UTILIZED
C                   PRIOR TO V MIN.  AFTER V MIN THE ELEVATOR ANGLE
C                   IS GRADUALLY RETURNED TO THE ELEVATOR ANGLE AT TRIM.
C
C
C        I4         THRUST OPTIONS DURING THE STALL MANEUVER
C
C
C        VALUE                          OPTION
C
C
C         1         THE VALUE OF THRUST SELECTED BY I1 WILL CONTINUE
C                   TO APPLY AFTER V MIN
C
C         2         THE VALUE OF THRUST SELECTED BY I1 WILL BE INCREASED
```

```
C                TO TRIM THRUST (I1=3) OR INCREASED BY (TRIM THRUST-
C                250 LB.) OVER A 1.2 SECOND PERIOD
C
C
C     EXPONENTIAL ELEVATOR SCHEDULE:
C
C
C     FOR THE EXPONENTIAL ELEVATOR SCHEDULE I3=2 AND VALUES FOR
C     I2=1 THRU 9 ARE USED.  WHEN THE AIRCRAFT DECELERATION RATE
C     FALLS BELOW THE DESIRED RATE (DENOTED AS NUM1 AND IN KTS/SEC)
C     THE PROGRAM AUTOMATICALLY SWITCHES TO AN EXPONENTIAL
C     ELEVATOR SCHEDULE.  THE ORGINALLY SELECTED LINEAR SCHEDULE
C     IS MULTIPLIED BY E RAISED TO THE RI POWER.  THE LINEAR VALUES
C     ARE THE SAME AS DEFINED BY I2.  THE DESIRED EXPONENTIAL
C     RATE (RI) MUST BE INSERTED INTO THE PROGRAM PRIOR TO
C     EXECUTION.
C
C
      IMPLICIT REAL*8(A-H,K-Z)
      REAL*4 RANGE,RANGE1,RANGE2,RANGE3,RANGE4,RANGE5,RANGE6
C
C
C     THE FUNCTION SUBROUTINES UTILIZED BY SUBROUTINE RUNKUT ARE
C     DEFINED AS:
C
C
C     FUNCTION F1 = U-DOT
C     FUNCTION F2 = W-DOT
C     FUNCTION F3 = THETA-DOT
C     FUNCTION F4 = THETA-DBL. DOT
C     FUNCTION F5 = ALPHA-DOT
C
C
C
C     THE FOLLOWING DIMENSIONED QUANTITIES ARE:
C
C
C     T(I)=REAL TIME
C     X1(I)=U VELOCITY COMPONENT
C     X2(I)=W VELOCITY COMPONENT
C     X3(I)=PITCH ANGLE THETA IN RADIANS
C     X4(I)=RATE OF CHANGE OF PITCH ANGLE (THETA DOT IN RADIANS)
C     X5(I)=ANGLE OF ATTACK ALPHA IN RADIANS
C     H(I)=ALTITUDE IN FEET
C     HDOT(I)=RATE OF CHANGE ALTITUDE IN FT. PER MINUTE
C     ANFP(I)=FLIGHT PATH NORMAL ACCELERATION
C     ACL(I)=AIRCRAFT LIFT COEFFICIENT
C     ACD(I)=AIRCRAFT DRAG COEFFICIENT
```

```
C     ACM(I)=AIRCRAFT PITCHING MOMENT COEFFICIENT
C     DEL(I)=ELEVATOR ANGLE
C     CLP(I)=APPARENT AIRCRAFT LIFT COEFFICIENT
C     VEL(I)=AIRCRAFT RESULTANT VELOCITY
C
C
      DIMENSION T(641),X1(641),X2(641),X3(641),X4(641),X5(641),VEL(641),
     1ANFP(641),H(641),ACL(641),ACD(641),ACM(641),DEL(641),CLP(641),RANG
     2E(4),RANGE1(4),RANGE2(4),RANGE3(4),RANGE4(4),RANGE5(4),RANGE6(4)
C
C
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAC,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
C
      READ(5,37) I1,I2,I3,I4
      READ(5,1) TT,CLTD,CLDE,RHO,S,MA,JM
      READ(5,2) CMTD,CMAD,CMDE,BIY,CBAR
C
C
C     THE FOLLOWING ARRAYS ARE READ INTO THE PROGRAM TO BE USED BY
C     SUBROUTINE SPLIN1.  SPLIN1 IS GIVEN 14 SETS VALUES TO BE USED
C     AS ORDINATE AND ABASSIA FOR A TABLE LOOK UP.  SPLIN1 TAKES A
C     A GIVEN VALUE OF ANGLE OF ATTACK AND PROVIDES THE NON-LINEARITY
C     IN AIRCRAFT CL, CD AND CM.  THE ARRAYS ARE:
C
C     X=VALUES OF ANGLE CF ATTACK
C     Y=VALUES OF LIFT COEFFICIENT
C     V=VALUES OF DRAG COEFFICIENTS
C     XX=VALUES OF PITCHING MOMENT COEFFICIENT
C
C     SPLIN1 TAKES A GIVEN VALUE OF ANGLE OF ATTACK AND PROVIDES THE
C     CORESPONDING VALUE OF CL,CD OR CM
C
C
      READ(5,3) X
      READ(5,3) Y
      READ(5,3) V
      READ(5,3) XX
C
C
C     STEP SIZE HI USED IN RUNGA-KUTTA SCHEME IS IN REAL TIME SECONDS.
C     JJ, JJJ, IJ, J1 ARE INTEGERS USED IN PROGRAMING THE DESIRED
C     RATE OF CHANGE OF ELEVATOR ANGLE.
C
C
      HI=0.5D-01
      G=32.174
      JJ=1
```

```
      JJJ=3
      NUM1=2.15
      R1=0.0
      IM=641
      IMM=IM-1
      IJ=40
      II=1
      J1=1
C
C
C     THE DENSITY ALTITUDE IS SEA LEVEL, BUT THE AIRCRAFT WAS
C     ARBITRARILY PLACED AT 5M FEET TO AVOID NEGATIVE ALTITUDES.
C
C
      H(1)=5000
C
C
C     THE FOLLOWING VALUES OF RANGE DEFINE THE SCALES FOR THE FOUR
C     OUTPUT GRAPHS.
C
C
      RANGE(1)=32.0
      RANGE(2)=0.0
      RANGE(3)=129.0
      RANGE(4)=87.0
      RANGE1(1)=32.0
      RANGE1(2)=0.0
      RANGE1(3)=7000.0
      RANGE1(4)=1000.0
      RANGE2(1)=32.0
      RANGE2(2)=0.0
      RANGE2(3)=2.0
      RANGE2(4)=0.8
      RANGE3(1)=32.0
      RANGE3(2)=0.0
      RANGE3(3)=2.0
      RANGE3(4)=0.8
      RANGE4(1)=32.0
      RANGE4(2)=0.0
      RANGE4(3)=32.0
      RANGE4(4)=2.0
      RANGE5(1)=32.0
      RANGE5(2)=0.0
      RANGE5(3)=18.0
      RANGE5(4)=0.0
      RANGE6(1)=32.0
      RANGE6(2)=0.0
      RANGE6(3)=1.6
```

```
      RANGE6(4)=0.4
C
C
C     THE INITIAL CONDITIONS ARE READ INTO THE PROGRAM.
C
C
      T(1)=0.0D0
      READ(5,4) W,AOA,THETA
C
C
C     ALL VALUES READ INTO THE PROGRAM ARE PRINTED OUT AS AN ECHO CHECK.
C
C
      WRITE(6,31)
      WRITE(6,5) TT,CLTD,CLDE,RHO,S,MA,JM,CMTD,CMAD,CMDE,BIY,CBAR,I1,
     1I2,I3,I4
      WRITE(6,6)
      WRITE(6,7)  (X(I),Y(I),V(I),XX(I), I=1,14)
      ATH=THETA-AOA
      IF(DABS(ATH).GT.0.1D-02) GO TO 9
      WRITE(6,8) AOA,W
C
C
C
C     THE INPUT DATA IS USED TO CALCULATE THE TRIM CONDITIONS FOR
C     STEADY LEVEL FLIGHT.  SUBROUTINE TRIM TAKES ANGLE OF ATTACK PLUS
C     THE OTHER AIRCRAFT DATA AND CALCULATES THE CORRESPONDING VELOCITY,
C     ELEVATOR ANGLE, THRUST,CL,CD,AND CM.  SUBROUTINE TRIM USES THE
C     FORCE AND MOMENT EQUATIONS AT STATIC EQUILIBRIUM.  THIS CALCULATED
C     DATA IS THEN UNED AS THE INITIAL CONDITIONS TO START THE RUNKA-
C     DUTTA ITERATIVE SCHEME.
C
C
C
C
11    CALL TRIM(W,THETA,AOA,B1,C1,D1,E1,FF,VLL,I1)
      WRITE(6,17)
      WRITE(6,18) B1,C1,VLL,D1,E1,FF,DE,CL,CM,CD,TT
      X1(1)=B1
      X2(1)=C1
      X3(1)=D1
      X4(1)=E1
      X5(1)=FF
      DEL(1)=DE
      ACL(1)=CL
      ACM(1)=CM
      ACD(1)=CD
      A=T(1)
      B=X1(1)
```

```
      C=X2(1)
      D=X3(1)
      E=X4(1)
      F=X5(1)
      WRITE(6,21)
C
C
C     THE PROGRAM NOW PROVIDES THE DESIRED THRUST OPTION.
C
C
      DELTT=(TT-250.0)/24.0
      IF(DELTT) 101,101,102
101   DELTT=0.0
102   DO 20 I=1,IMM
      IF(I.GE.25) GO TO 14
      GO TO (14,12,13,19), I1
12    TT=0.0
      GC TO 14
13    TT=TT-DELTT
14    IF(I4.EQ.1) GO TO 19
      IF(I.LE.80) GO TO 19
      BVEL=DSQRT(X1(I-1)*X1(I-1)+X2(I-1)*X2(I-1))
      CVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
      XVEL=BVEL-CVEL
      IF(XVEL) 15,19,19
15    IF(I4.GE.3) GO TO 16
      I4=I+60
16    IF(I.LT.I4) GO TO 19
      IF(JJ.GE.25) GO TO 19
      TT=TT+DELTT
      JJ=JJ+1
C
C
C     SUBROUNTINE RUNKUT USES THE TRIM VALUES CALCULATED BY SUBROUTINE
C     TRIM AS INITIAL CONDITIONS TO START THE ALGORITHM.  THESE
C     TRIM VALUES CORRESPCND TO TIME T(1)=0.  VALUES RETURNED BY
C     RUNKUT ARE FOR THE NEXT INCREMENT IN TIME.  THESE VALUES
C     ARE THEN USED AS INITIAL CONDITIONS FOR CALCULATING THE
C     AIRCRAFT PARAMETERS AT THE NEXT TIME STEP.  THEREFORE, ONCE
C     STARTED, THE RUNGE-KUTTA ALGORITHM IS SELF SUSTAINING.
C
C
19    CALL RUNKUT(A,B,C,D,E,F,B1,C1,D1,E1,FF)
      X1(I+1)=B1
      X2(I+1)=C1
      X3(I+1)=D1
      X4(I+1)=E1
      X5(I+1)=FF
```

```
      T(I+1)=T(I)+HI
      A=T(I+1)
      B=X1(I+1)
      C=X2(I+1)
      D=X3(I+1)
      E=X4(I+1)
      F=X5(I+1)
      DEL(I+1)=DE
      ACL(I+1)=CL
      ACM(I+1)=CM
      ACD(I+1)=CD
C
C
C     THE PROGRAM NOW PROVIDES THE DESIRED ELEVATOR SCHEDULE THAT
C     IS CHOSEN.
C
C
      IF(DABS(DEL(I)).GE.0.28) GO TO 20
      IF(I3.GE.2) GO TO  57
      GO TO (23,23,24,24,20,25,25,25,25), I2
23    DE=DE-(0.1D-03*I2)
      GO TO 20
24    DE=DE-(0.15D-03+(I2-3)*(0.1D-03))
      GO TO 20
25    IF(II.GE.IJ) GO TO 20
      IF(I.LE.40) GO TO 54
      BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
      CVEL=DSQRT(X1(I+1)*X1(I+1)+X2(I+1)*X2(I+1))
      XVEL=BVEL-CVEL
      IF(XVEL) 51,51,54
51    II=II+1
54    IF(I2-8) 55,56,56
55    DE=DE-(0.1D-03*(I2-5))
      GO TO 20
56    DE=DE-(0.15D-03+(I2-8)*(0.1D-03))
      GO TO 20
57    GO TO (20,80,80,58),I3
58    IF(II.GE.IJ) GO TO 59
      IF(I.LE.40) GO TO 54
      BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
      CVEL=DSQRT(X1(I+1)*X1(I+1)+X2(I+1)*X2(I+1))
      XVEL=BVEL-CVEL
      IF(XVEL) 51,54,54
59    DE=DE+0.1D-03
      IF(DEL(1)-DE) 60,60,20
60    DE=DEL(1)
      GO TO 20
80    IF(J1.GE.IJ) GO TO 20
```

```
      IF(I.LE.40) GO TO 86
      BVEL=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))
      CVEL=DSQRT(B1*B1+C1*C1)
      XVEL=BVEL-CVEL
      IF(XVEL) 81,81,82
81    J1=J1+1
82    DVEL=(XVEL/0.05)*0.592086
      IF(DVEL-NUM1) 83,86,86
83    GO TO (20,84,85,58), I3
84    R1=R1+0.002
      DE=DE-(0.1D-03*I2)*DEXP(R1)
      GO TO 20
85    R1=R1+0.002
      DE=DE-(0.15D-03+(I2-3)*(0.1D-03))*DEXP(R1)
      GO TO 20
86    GO TO (20,23,24,58), I3
20    CONTINUE
C
C
C     THE RESULTANT AIRCRAFT VELOCITY, NORMAL FLIGHT PATH ACCELERATION,
C     APPARENT LIFT COEFFICIENT AND ALTITUDE IS CALCULATED FOR EACH
C     TIME STEP.
C
C
      DO 30 I=1,IM
      VEL(I)=DSQRT(X1(I)*X1(I)+X2(I)*X2(I))*0.592086
      ANFP(I)=((VEL(I)*X4(I))/G)+DCOS(X3(I)-X5(I))
      CLP(I)=ACL(I)/ANFP(I)
      HDOT=VEL(I)*DSIN(X3(I)-X5(I))*101.3364
      IF(I.EQ.IM) GO TO 30
      H(I+1)=H(I)+(HDOT*(T(I+1)-T(I))/60.0)
30    CONTINUE
      J=75
      DO 41 I=1,IM
      IF(I.EQ.J) GO TO 42
40    WRITE(6,22)  T(I),X1(I),X2(I),VEL(I),X3(I),X4(I),X5(I),DEL(I),ACL(
     1I),CLP(I),ACM(I),ACD(I),ANFP(I),H(I)
41    CONTINUE
      GO TO 36
42    WRITE(6,21)
      J=J+75
      GO TO 40
36    WRITE(6,32)
      DO 50 I=1,IM
      X5(I)=X5(I)*57.29577951
      DEL(I)=DEL(I)*57.29577951*(-1.0)
50    CONTINUE
      CALL UTPLOT(T,VEL,641,RANGE,2,0)
```

```
      WRITE(6,33)
      CALL UTPLOT(T,H,641,RANGE1,2,0)
      WRITE(6,34)
      CALL UTPLOT(T,ACL,641,RANGE2,2,0)
      WRITE(6,35)
      CALL UTPLOT(T,CLP,641,RANGE3,2,0)
      WRITE(6,71)
      CALL UTPLOT(T,X5,641,RANGE4,2,0)
      WRITE(6,72)
      CALL UTPLOT(T,DEL,641,RANGE5,2,0)
      WRITE(6,73)
      CALL UTPLOT(T,ANFP,641,RANGE6,2,0)
      GO TO 100
9     WRITE(6,10) AOA,THETA,W
      GO TO 11
1     FORMAT(F10.3,5F10.5,I3)
2     FORMAT(3F10.5,F10.2,F10.5)
3     FORMAT(7F10.7)
4     FORMAT(F10.2,F10.6,F10.6)
5     FORMAT(T55,'INPUT DATA',//,T20,'TT    =',T26,F10.3,T40,'CLTD =',T46
     1,F9.6,T60,'CLDE =',T66,F7.4,T80,'RHO   =',T86,F10.7,//,T20,'S    ='
     2,T26,F7.2,T40,'MASS =',T46,F7.2,T60,'JM    =',T66,I3,T80,'CMTD =',T
     386,F9.6,//,T20,'CMAD =',T26,F9.6,T40,'CMDE =',T46,F7.4,T60,'BIY  =
     4',T66,F9.2,T80,'CBAR =',T86,F5.2,//,T20,'I1    =',T26,I2,T40,'I2
     5=',T46,I2,T60,'I3    =',T66,I2,T80,'I4    =',T86,I2)
6     FORMAT(//,T51,'DATA USED BY SPLIN IN TABLE LOOK UP',//,T18,'ANGLE
     1OF ATTACK',T55,'CL',T85,'CD',T114,'CM',//)
7     FORMAT('0',T18,F12.6,T48,F12.6,T78,F12.6,T108,F12.6)
8     FORMAT(///,T40,'INPUT DATA',//,T20,'ANGLE OF ATTACK = PITCH ANGLE
     1=',T52,F8.5,T65,'(AIRCRAFT IN LEVEL FLIGHT)',//,T20,'AIRCRAFT WEIG
     2HT = ',T38,F9.2)
10    FORMAT(///,T61,'INPUT DATA',//,T52,'ANGLE OF ATTACK =',T70,F8.5,
     1//,T52,'PITCH ANGLE =',T70,F8.5,//,T52,'AIRCRAFT WEIGHT =',T70,
     2F9.2)
17    FORMAT(////,T18,'FOR TRIMMED LEVEL FLIGHT, THE AIRCRAFT PARAMETERS
     1 HAVE THE FOLLOWING VALUES.',//,T3,'X VEL',T14,'Z VEL',T23,
     2'A/C VEL',
     3         T38,'THETA',T50,'THETA DOT',T66,'AOA',T75,'ELEV. ANGLE',T9
     44,'CL',T108,'CM',T122,'CD',//)
18    FORMAT(T1,F8.3,T12,F8.3,T23,F8.3,
     1                        T33,F12.4,T47,F10.5,T61,F10.5,T75,F10.7,
     2T89,F10.7,T103,F10.7,T117,F10.7,///,T38,'THRUST FOR LEVEL FLIGHT =
     3',T64,F8.2)
21    FORMAT ('1',T2,'TIME',T12,'X VEL',T22,'Z VEL',T31,'A/C VEL',T43,'T
     1HETA',T50,'THETA DOT',T64,'AOA',T74,'DE',T85,'CL',T92,'CL-*',T101,
     2'CM',T111,'CD',T119,'ANFP',T128,'ALT',//)
22    FORMAT(F7.3,T10,F8.3,T20,F8.3,T30,F8.3,T41,F8.5,T51,F8.5,T61,F8.5,
     1T71,F8.5,T81,F7.4,T90,F7.4,T99,F7.4,T108,F7.4,T117,F7.4,T126,F7.1)
```

```
31    FORMAT('1')
32    FORMAT('1',///,T34,'RESULTANT AIRCRAFT VELCCITY(KTS.) VS. TIME',//
     1/)
33    FCRMAT('1',///,T41,'AIRCRAFT ALTITUDE VS. TIME',////)
34    FORMAT('1',///,T43,'AIRCRAFT  CL  VS. TIME',////)
35    FORMAT('1',///,T30,'AIRCRAFT CL-* (BASED ON NORMAL FLIGHT PATH ACC
     1EL.) VS. TIME',////)
37    FORMAT(4I2)
71    FORMAT('1',///,T36,'AIRCRAFT ANGLE OF ATTACK (DEG) VS. TIME',///)
72    FORMAT('1',///,T39,'ELEVATOR ANGLE (T.E. UP) VS. TIME',///)
73    FORMAT('1',///,T35,'NORMAL FLIGHT ACCELERATION VS. TIME',///)
100   STOP
      END

      SUBROUTINE RUNKUT(A,B,C,D,E,F,B1,C1,D1,E1,FF)
C
C
C     SUBROUTINE RUNKUT EMPLOYES A FOURTH-ORDER RUNGE-KUTTA
C     ALGORITHM IN SOLVING THE NON-LINEAR EQUATIONS OF AIRCRAFT
C     MOTION.  RUNKUT CALLS FUNCTION SUBROUTINES F1, F2,F3, F4 AND
C     F5 TO CALCULATE THE DERIVATIVE TERMS AT INTERMEDIATE TIME
C     STEPS.  THE NON-LINEARITIES IN THE AERODYNAMIC DATA IS
C     INCORPORATED IN F1 AND IS OBTAINED FROM TABLE LOOK-UP BY USING
C     A CUBIC SPLINE FUNCTION, SUBROUTINE SPLIN1.  INPUT DATA USED
C     BY SPLIN1 IN THE TABLE LOOK-UP PROCEDURE ARE LISTED IN TABLE I.
C
C
      IMPLICIT REAL*8(A-H,K-Z)
      HI=0.5D-01
      K1=HI*F1(A,B,C,D,E,F)
      L1=HI*F2(A,B,C,D,E,F)
      M1=HI*F3(A,B,C,D,E,F)
      N1=HI*F4(A,B,C,D,E,F)
      P1=HI*F5(A,B,C,D,E,F)
      K2=HI*F1(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,F+P1/2.0)
      L2=HI*F2(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,F+P1/2.0)
      M2=HI*F3(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,F+P1/2.0)
      N2=HI*F4(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,F+P1/2.0)
      P2=HI*F5(A+HI/2.0,B+K1/2.0,C+L1/2.0,D+M1/2.0,E+N1/2.0,F+P1/2.0)
      K3=HI*F1(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,F+P2/2.0)
      L3=HI*F2(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,F+P2/2.0)
      M3=HI*F3(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,F+P2/2.0)
      N3=HI*F4(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,F+P2/2.0)
      P3=HI*F5(A+HI/2.0,B+K2/2.0,C+L2/2.0,D+M2/2.0,E+N2/2.0,F+P2/2.0)
      K4=HI*F1(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
      L4=HI*F2(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
      M4=HI*F3(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
```

```
      N4=HI*F4(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
      P4=HI*F5(A+HI,B+K3,C+L3,D+M3,E+N3,F+P3)
      B1=B+(K1+2.0*K2+2.0*K3+K4)/6.0
      C1=C+(L1+2.0*L2+2.0*L3+L4)/6.0
      D1=D+(M1+2.0*M2+2.0*M3+M4)/6.0
      E1=E+(N1+2.0*N2+2.0*N3+N4)/6.0
      FF=F+(P1+2.0*P2+2.0*P3+P4)/6.0
      RETURN
      END

      FUNCTION F1(T,X1,X2,X3,X4,X5)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
      AA=(X1*X1+X2*X2)
      CALL SPLIN1(X,Y,JM,X5,CLA)
      CALL SPLIN1(X,XX,JM,X5,CMA)
      CALL SPLIN1(X,V,JM,X5,CD)
      CT=(2.0*TT)/(RHO*AA*S)
      CL=CLA+CLTD*0.5*CBAR*X4/DSQRT(AA)+CLDE*DE
      CX=CT+CL*DSIN(X5)-CD*DCOS(X5)
      CZ=(-1.0*(CL*DCOS(X5)+CD*DSIN(X5)))
      F1=((RHO*AA*S)/(2.0*MA))*CX-G*DSIN(X3)-X2*X4
      Z1=F1
10    RETURN
      END

      FUNCTION F2(T,X1,X2,X3,X4,X5)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
      F2=((RHO*AA*S)/(2.0*MA))*CZ+G*DCOS(X3)+X1*X4
      Z2=F2
10    RETURN
      END

      FUNCTION F3(T,X1,X2,X3,X4,X5)
      IMPLICIT REAL*8(A-H,K-Z)
      F3=X4
10    RETURN
      END
```

```
      FUNCTION F4(T,X1,X2,X3,X4,X5)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
      CM=CMA+(CMTD*0.5*CBAR*X4)/DSQRT(AA)+CMAD*((Z2*DCOS(X5)-Z1*DSIN(X5)
     1)*0.5*CBAR/AA)+CMDE*DE
      F4=((RHO*AA*S*CBAR)/(2.0*BIY))*CM
   10 RETURN
      END


      FUNCTION F5(T,X1,X2,X3,X4,X5)
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
      F5=(Z2*DCOS(X5)-Z1*DSIN(X5))/DSQRT(AA)
   10 RETURN
      END


      SUBROUTINE TRIM(W,THETA,AOA,B1,C1,D1,E1,FF,VLL,I1)
C
C
C     SUBROUTINE TRIM UTILIZES THE STATIC AIRCRAFT LONGITUDINAL
C     MOMENT AND FORCE EQUATIONS IN TRIMMING THE AIRCRAFT IN THE
C     DESIRED ATTITUDE.  DESIRED ANGLE OF ATTACK, PITCH ANGLE AND
C     AIRCRAFT WEIGHT ARE INPUT INTO TRIM.  THE AIRCRAFT CAN BE IN
C     A CLIMBING, LEVEL OR DESCENDING FLIGHT ATTITUDE.  TRIM THEN
C     CALCULATES TRIM VALUES FOR CL, CD, CM, ELEVATOR ANGLE AND
C     THRUST.  THESE VALUES ARE THEN PASSED TO THE MAIN PROGRAM TO
C     BE USED AS INITIAL CONDITIONS FOR STARTING THE RUNGE-KUTTA
C     ALGORITHM.
C
C
      IMPLICIT REAL*8(A-H,K-Z)
      COMMON CM,CL,CD,CZ,DE,BIY,CMTD,CMAD,CMDE,CBAR,V(14),Z1,Z2,XX(14),A
     1A,CLTD,CLDE,CT,G,RHO,S,X(14),Y(14),MA,CMA,TT,JM
      X5=AOA
      CM=0.0
      CALL SPLIN1(X,Y,JM,X5,CLA)
      CALL SPLIN1(X,XX,JM,X5,CMA)
      CALL SPLIN1(X,V,JM,X5,CD)
      DE=-1.0*(CMA/CMDE)
      CL=CLA+CLDE*DE
      AT=THETA-AOA
      IF(AT) 1,2,2
    1 AT=AOA-THETA
    2 IF(I1.EQ.2) GO TO 3
```

```
      IF(I1.EQ.4) GO TO 4
      SAT=DSIN(AT)
      CAT=DCOS(AT)
      SA=DSIN(AOA)
      CA=DCOS(AOA)
      NUM1=W*(CAT*CA-SAT*SA)
      DEN1=S*(CL*CA+CD*SA)
      Q=NUM1/DEN1
      LD=CL/CD
      NUM2=W*(LD*SAT+CAT)
      DEN2=LD*CA+SA
      TT=NUM2/DEN2
      GO TO 7
3     Q=(W*DCOS(AT))/(CL*S)
      TT=0.0
      GO TO 7
4     TT=W*0.019
      A=(S*S)*((CL*CL)+(CD*CD))
      B=2.0*S*TT*(CL*DSIN(X5)-CD*DCOS(X5))
      C=((TT*TT)-(W*W))
      BB=(B/(2.0*A))
      Q=-BB+DSQRT((BB*BB)-(C/A))
      TA=DARCOS((CL*S*C+TT*DSIN(X5))/W)
5     THETA=X5-TA
7     VX=(2.0*Q)/RHO
      VLL=DSQRT(VX)
      B1=VLL*DCOS(X5)
      C1=VLL*DSIN(X5)
      D1=THETA
      E1=0.0
      FF=X5
      RETURN
      END


      SUBROUTINE SPLIN1(X,Y,M,XINT,YINT)
      IMPLICIT REAL*8 (A-H);REAL*8 (O-Z)
      DIMENSION X(M),Y(M),C(4,30)
      CALL SPLICO(X,Y,M,C)
      K=1
      ENTRY SPLINN(X,Y,M,XINT,YINT)
    3 IF(XINT-X(1)) 70,1,2
   70 K=1
      GO TO 7
    1 YINT=Y(1)
      RETURN
    2 IF(XINT-X(K+1))6,4,5
    4 YINT=Y(K+1)
```

```
      RETURN
    5 K=K+1
      IF(M-K) 71,71,3
   71 K=M-1
      GO TO 7
    6 IF(XINT-X(K))13,12,7
   12 YINT=Y(K)
      RETURN
   13 K=K-1
      GO TO 6
    7 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))
      YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))
      RETURN
      END


      SUBROUTINE SPLICO(X,Y,M,C)
      IMPLICIT REAL*8 (A-H),REAL*8 (O-Z)
      DIMENSION X(M),Y(M),C(4,30),D(30),P(30),E(30),A(30,3),B(30),Z(30)
      MM=M-1
      DO 2 K=1,MM
      D(K)=X(K+1)-X(K)
      P(K)=D(K)/6.
    2 E(K)=(Y(K+1)-Y(K))/D(K)
      DO 3 K=2,MM
    3 B(K)=E(K)-E(K-1)
      A(1,2)=-1.-D(1)/D(2)
      A(1,3)=D(1)/D(2)
      A(2,3)=P(2)-P(1)*A(1,3)
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)
      A(2,3)=A(2,3)/A(2,2)
      B(2)=B(2)/A(2,2)
      DO 4 K=3,MM
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)
      B(K)=B(K)-P(K-1)*B(K-1)
      A(K,3)=P(K)/A(K,2)
    4 B(K)=B(K)/A(K,2)
      Q=D(M-2)/D(M-1)
      A(M,1)=1.+Q+A(M-2,3)
      A(M,2)=-Q-A(M,1)*A(M-1,3)
      B(M)=B(M-2)-A(M,1)*B(M-1)
      Z(M)=B(M)/A(M,2)
      MN=M-2
      DO 6 I=1,MN
      K=M-I
    6 Z(K)=B(K)-A(K,3)*Z(K+1)
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
      DO 7 K=1,MM
```

```
      Q=1./(6.*D(K))
      C(1,K)=Z(K)*Q
      C(2,K)=Z(K+1)*Q
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)
    7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
      RETURN
      END
```

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Evaluation of Minimum Aircraft Flying Speed by Digital Simulation

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis (June 1972)

5. AUTHOR(S) *(First name, middle initial, last name)*

Carlton Wayne Saul

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1972	73	8

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Aircraft minimum flying speed, as determined by actual flight test, is published in aircraft handbooks for pilot guidance. The test flight results are used to determine and confirm take-off and landing speeds, field lengths, left-hand portion of the maneuvering envelopes (V-n diagram), etc. Determination of the absolute minimum flying speed of an aircraft on the other hand, has not been of prime importance in flight test.

In the present analysis digital simulation allowed the systematic study of not only the minimum flying speed as defined by Federal Aviation Regulations but also the absolute minimum flying speed attainable in steady, unaccelerated flight. The study included such effects as deceleration rate, rate of change of elevator angle, aircraft weight and pitch moment of inertia.

It was found for an assumed light-weight fighter aircraft that the absolute minimum flying speed was approximately 20 knots less than the FAR minimum flying speed. Moreover the FAR minimum flying speeds tended to be quite sensitive to rate of change of elevator angle.

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S/N 0101-807-6811

Security Classification

A-31408

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Absolute Minimum Flying Speed						
Stall Speed						
FAR Minimum Flying Speed						
Certified Minimum Flying Speed						
Non-Linear Equations of Aircraft Motion						
Digital Simulation of Aircraft Motion						
Fourth-Order Runge-Kutte Algorithm						
F-94A Stall Characteristics						

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Evaluation of minimum
aircraft flying speed
by digital simulation.